

Unit Vectors and the Cross Product:

Problems on Quest almost always depend on unit vectors, and cross products of unit vectors. Here is a brief summary. The definition of a vector cross product $\mathbf{A} \times \mathbf{B}$ is that it results in a vector \mathbf{C} which is at right angles to the plane containing the two crossed vectors, and has magnitude $C = AB \sin \theta$, where θ is the angle between \mathbf{A} and \mathbf{B} . The direction of \mathbf{C} is given by a right hand rule: start with the fingers of the right hand aligned with \mathbf{A} , and sweep them around so that they align with \mathbf{B} . Then your thumb will point along \mathbf{C} .

Now see how this works with unit vectors, which are vectors of length 1 that point along x , y and z . They are correctly written as $\hat{\mathbf{i}}$ (along x), $\hat{\mathbf{j}}$ (along y) and $\hat{\mathbf{k}}$, along z . Using the definition of cross product we just gave, you can instantly see by looking at your right hand that

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}.$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}.$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}.$$

Note that because $\mathbf{B} \times \mathbf{A} = -\mathbf{C}$, you can cross the unit vectors “backward,” getting a minus sign (for example $\hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}}$).