SCATTERING!

The main way we learn about what surrounds us is via the scattering of light (photons) from the objects around us. The part of the scattered light that enters our eyes allows us to form an image of the objects. Physicists use scattering experiments to learn about the positions of atoms relative to one another in solids, the internal structure of atoms and molecules, and even the internal structure of protons. One key part of the description of scattering is the total cross section $\sigma$, which roughly speaking is the area through which particles in a beam would have to pass in order to interact with target particles. For a force of infinite range, the total cross section is infinite, for obvious reasons. However, for two of the fundamental forces of nature, the strong and weak forces, the effective range is very short, and finite total cross sections exist and can be easily measured. Whether the force has infinite or finite range, the most useful quantity to measure is the differential cross section or angular distribution, usually written $d\sigma/d\Omega$. The usual unit of $\sigma$ is the barn, which is $10^{-28}$ m$^2$. A more convenient unit of distance is the fm, or Fermi, which is $10^{-15}$ m, so a barn is 100 fm$^2$. By historical accident, a barn is a bit too large to be convenient,
so the usual unit is the millibarn, mb, which is $10^{-3}$ barns. The usual unit for the differential cross section, therefore, is mb per steradian, where the solid angle $\Omega = A/r^2$ is subtended by an area $A$ at a distance of $r$. In the text this concept is avoided, and the differential cross section is generally obtained in the form $d\sigma/d\cos\theta$, where $\theta$ is the scattering angle. This quantity just has units of area, in other words mb.

Using classical physics and the Coulomb force magnitude $F = (k2Ze^2)/r^2$ where $Ze$ is the charge on the nucleus and $2e$ is the charge on the $\alpha$ particle, Rutherford derived

$$\frac{d\sigma}{d\cos\theta} \propto \left(\frac{2Ze^2}{K}\right)^2 \frac{1}{1 - \cos\theta}.$$

Here $K$ is the kinetic energy of the projectile. The fundamental coupling constant for electromagnetic processes is called $\alpha$ (don’t confuse with the symbol for the alpha particle!) and equals $ke^2/(\hbar c)$. This is basically the probability that a virtual photon is emitted by one charge $e$ and absorbed by another. Note that $\hbar c$ in convenient units is 197 MeV-fm. Then the Rutherford cross section is

$$\frac{d\sigma}{d\cos\theta} \propto \alpha^2 \left(\frac{\hbar c}{K}\right)^2 \frac{1}{1 - \cos\theta}.$$
Learning about the nucleus from scattering: In the experiments done at Rutherford’s lab, there was obtained no information about the size of the nucleus, since the kinetic energy of the α particles was just a few MeV and the resulting probability wavelength $\lambda = h/p$ was huge compared to the actual nuclear radius, so that there was no observable diffraction of the probability wave. The actual sizes of nuclei are a few fm, so we need a probe with a kinetic energy such that the probability wavelength is significantly less than a fm. Experiments done beginning in the 1950s, in which electrons of energies more than 1 GeV (10^3 MeV) were used to determine the nuclear charge distribution as a function of $A$, the number of nucleons (protons plus neutrons) in the nucleus, resulted in the discovery that the effective radius of the charge distribution of a nucleus is given fairly accurately by the expression $r = (1.2\text{fm}) \times A^{1/3}$. This result indicates that all nuclei have essentially the same density, namely $2.3 \times 10^{17} \text{ kg/m}^3$. Or $0.23 \times 10^{-27} \text{ kg per cubic fm}$. Remember a nucleon has a mass of about $1.7 \times 10^{-27} \text{ kg}$, so the density amounts to about 0.14 nucleons per cubic fermi. The volume of a single nucleon is roughly 2 fm$^3$, for comparison. The typical distance between nucleons in a nucleus is roughly 2 fm.
Protons and neutrons each have a size of about 0.8 fm, so with electron beams of appropriately large kinetic energy, so that the probability wavelengths are considerably less than 0.1 fm, one can in elastic scattering experiments measure the charge distributions of the proton and the neutron! People then naturally began to ask, if the proton and neutron were to turn out to be bound states of pointlike constituents, what are those constituents? By 1970 the answer had become very clear, as electron energies from accelerators became large enough so that the electrons could scatter off pointlike constituent particles inside the proton!

The experiments confirmed that protons and neutrons are indeed made of pointlike fundamental particles, called quarks. The quarks have very small masses, of the order of an MeV. They interact by exchanging the bosons of the strong interaction, called gluons. There are 6 different quarks all told, but the “real” ones inside the proton and neutron are called up and down (u and d) quarks. The up quark has a charge of \(+\frac{2}{3}e\), where \(e\) is the magnitude of the electron charge, and the down quark has a charge of \(-\frac{1}{3}e\). The proton consists of 2 ups and a down, resulting in a total charge of \(\frac{4}{3} - \)
1/3)e = e. The neutron consists of 2 downs and an up, resulting in a total charge of \((-2/3 + 2/3)e = 0\). In addition to many virtual gluons, the proton and neutron are also found to contain a huge number of virtual pairs of quark and antiquark; all six possible pairs are found. The real quarks are therefore called "valence" quarks, while the virtual quarks are sometimes called "sea" quarks. [Antiparticles, a prediction of the Dirac Equation discovered in 1928, are partners to the standard particles. The antiparticle has exactly the same mass as the corresponding particle, but differs in additive quantum numbers such as charge. If a particle has no additive quantum numbers, it is its own antiparticle... an example being the photon. The electron's antiparticle is the positron, which had already been observed in radioactive decay without anyone knowing what it was, and the antiproton consists of 2 anti-up and 1 anti-down valence quarks, resulting in a total charge of \(-e\).]

If the quarks have masses of just a few MeV, how do the masses of the proton and neutron turn out to be close to 1 GeV? The answer is that almost all of this mass is \textit{internal energy}, the potential energy of the gluon field that holds the system together. [The glu-
ons, like the photon, are themselves massless.] Thus essentially all the mass of ordinary objects in the universe is due to the potential energy of the strong force holding nucleons together.