QUANTUM PHYSICS

(1) Every system, state and process in nature is described by an amplitude \( \psi(r, t) \) which assigns a complex number to every point in space-time.

(2) \( P(r, t) = |\psi(r, t)|^2 \) is the probability the process, state or system is found at the given point in space-time.

(3) For a particle with definite \( p \) and \( E \), the state function \( \psi(r, t) \) is wave-like, with frequency \( f = E/h \) and wavelength \( \lambda = h/p = h/\sqrt{2mE} \) (assuming a non-relativistic speed).

The universal constant \( h \) is \( 6.63 \times 10^{-34} \) J-s = \( 4.14 \times 10^{-15} \) eV-sec. The constant that occurs most often is \( \hbar = h/(2\pi) \).

Quantum systems cannot simultaneously possess a well-defined momentum and position, or a well-defined lifetime and total energy.

\[
\Delta x \Delta p_x \geq \frac{\hbar}{2}.
\]

\[
\Delta E \Delta t \geq \frac{\hbar}{2}.
\]

Thus a quantum system does not follow a trajectory or path in space. A classical trajectory has a well-defined \( p \) and \( r \) at each point in time. Such information does not exist in nature for quantum systems!
Particle with One Degree of Freedom Between Impenetrable Walls:

The probability amplitudes must be standing waves $\psi(x, t)$ going to zero at $x = 0$ and $x = L$. Thus the only allowed states have $\lambda_n = 2L/n$, where $n = 1, 2, 3$, etc.

This at once gives (for non-relativistic energies)

$$E_n = \frac{n^2 \hbar^2}{8mL^2} = n^2 E_1.$$  

Note these are the only possible energies, and the lowest possible energy is $E_1$, called the “ground state.”

When we plot $P(x, t) = |\psi(x, t)|^2$ we see interference! What is interfering? In quantum physics, whenever we have two indistinguishable possibilities, the states corresponding to both possibilities superpose and interfere. In this case the two interfering states have $p_x = p$ and $p_x = -p$. Note $\Delta p_x$ is $2p$ which is $2\sqrt{2mE}$. From the uncertainty relation, since $\Delta x = L$, we can see that $2L\sqrt{2mE} \geq \hbar/2$, which explains why $E$ is proportional to $1/L^2$ and to $\hbar^2$. 
Basic Rule of Quantum Physics:
If two or more processes are distinguishable in a given situation, and can happen simultaneously, the probabilities just add:

\[ P_{12\ldots} = |\psi_1|^2 + |\psi_2|^2 + \ldots \]

If two or more processes are indistinguishable in a given situation, and can happen simultaneously, the amplitudes superpose and interfere!

\[ P_{12\ldots} = |\psi_1 + \psi_2 + \ldots|^2. \]

The classic illustration of this is a double slit experiment done with quantum particles.

Barrier Penetration:
If a quantum particle encounters a potential barrier, it can penetrate it. In the simplest case of a rectangular barrier of height \( V \) and width \( d \), if the particle has \( E < V \), the probability of penetration is

\[ \mathcal{P} = \exp \left[ -2d \sqrt{2m(V-E)/\hbar^2} \right]. \]

The explanation is that in general the probability wavelength of the particle is significantly larger than
the width of the barrier, so the probability “sticks out the other side.”

A quantum particle can enter a region where its kinetic energy is negative, but in that region its probability amplitude is not wavelike, but rather is a steadily decreasing function. If the barrier has essentially infinite width, the probability eventually decreases to zero.
Two Families of Elementary Particles:

- **Fermions** are conserved in number, and no two identical fermions can occupy the same state.

- **Bosons** are not conserved in number, and can be created out of nothing. Any number of identical bosons can occupy precisely the same state.

**Matter** is made of fermions, which is why matter feels “solid” when fundamental fermions do not take up any space.

**Forces** are made of bosons, which can be emitted and absorbed to let fermions interact.

Example, repulsion of two identical charges $Q$. Physically, charge measures the probability of emitting or absorbing photons. This probability is

$$P = \frac{k_e Q^2}{\hbar c}.$$  

If the photon must exist long enough to cross a range $r$ it must have $\Delta p \approx \hbar / r$. But $r \approx c \Delta t$.

Thus

$$F = P \frac{\Delta p}{\Delta t} = \frac{k_e Q^2}{\hbar c} \frac{(\hbar / r)}{(r / c)}$$

which is

$$F = \frac{k_e Q^2}{r^2}$$

look familiar?

Bosons responsible for forces in this way are called “virtual bosons.”
Simple Questions:

- Imagine a nonrelativistic particle trapped in an impenetrable cubical box of volume $L^3$. What are its energy levels?

Answer:

$$E_{n_x,n_y,n_z} = (n_x^2 + n_y^2 + n_z^2) \frac{\hbar^2}{8mL^2}.$$ 

Any three-dimensional system will have three quantum numbers.

- A certain force acts through a virtual boson with a mass of 1000 MeV. Unlike gravity and electromagnetism, this force will have an extremely short range. Estimate it.

$Mc^2 \Delta t \simeq \hbar$, and $c\Delta t \simeq r_{\text{max}}$. So we instantly see that

$$r_{\text{max}} \simeq \frac{\hbar c}{Mc^2}.$$ 

Since $\hbar c$ is about 200 MeV-fm the result is $(200)/(1000) = 0.2$ fm! A fm is $10^{-15}$ m. For comparison the radius of a single proton is about 0.9 fm!