UNSTABLE NUCLEI:

It's a feature of quantum physics that transition probabilities are time-independent, and in such a case

$$N(t) = N(0)\exp(-t/\tau) = N(0)\left[\frac{1}{2}\right]^{t/T_{1/2}}.$$

Various processes are possible by which unstable nuclei can transform to nuclei of greater stability:

(1) Alpha decay— a ^{4}He cluster penetrates the Coulomb barrier.

(2) Beta-minus decay— a neutron converts into a proton, electron and anti-electron neutrino.

(3) Beta-plus decay— a proton converts into a neutron, positron and electron neutrino.

(4) Electron capture— a K electron is absorbed by a proton, yielding a neutron and an electron neutrino.

Processes (3) and (4) are always competitive except in very light nuclei.

(5) Spontaneous fission— in very heavy nuclei; because of the huge Coulomb barrier this process is not always probable.

(6) De-excitation of excited nuclear states, after decays (1) through (5):

(a) Gamma decay (emission of a photon).

(b) Internal conversion (ejected electron)— followed by characteristic X-rays or by Auger electrons.

As far as travelling across empty space, through various types of intervening matter, only gamma-ray photons need to be considered.

Radiation Units:

Source strength: 1 Becquerel = 1 decay/sec.

Absorbed dose: 1 Gray = 1 Joule/kg.

Actual effective dose: 1 Sievert. This conversion includes a weight factor to take account of the ionizing effectiveness of the projectile. It is about 1 for photons and leptons, about 5 for protons, and about 20 for α particles.

The usual radiation dose absorbed by humans is of the order of a few milliSievert per year.

Among the important natural sources are cosmic radiation, ^{40}K in the body, U and Th in dirt, and accumulations of ^{222}Rn, ^{220}Rn and other radioactive gases in enclosed spaces, particularly underground.

A linear extrapolation for the "lifetime risk of fatal cancer" due to cellular damage at the molecular level is around 5×10^{-2} per Sv. Actual observations of human populations exposed to various radiation levels, over the last 75 years, has suggested the actual risk is apparently orders of magnitude lower, leading to the conclusion that damage at this level has a strong threshold. [Life itself evolved under conditions of far, far higher natural radiation levels than we have today. The earliest traces of life date from around 4 billion years ago, appearing perhaps during the midst of the so-called Late Heavy Bombardment (4.1 to 3.8 billion years ago)!]

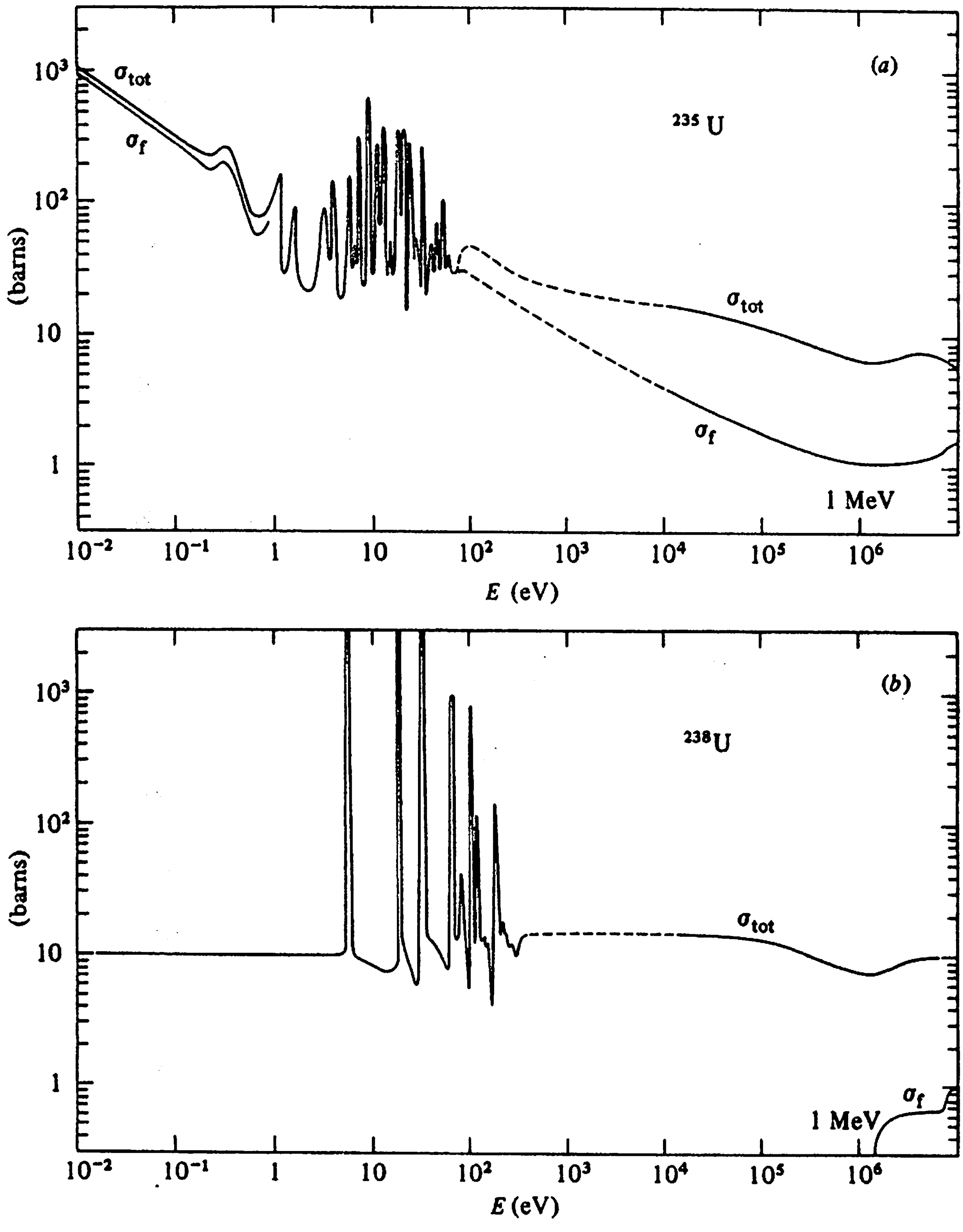


Fig. 9.1 Total cross-section σ_{tot} and fission cross-section σ_f as a function of energy for neutrons incident on (*a*) ^{235}U, (*b*) ^{238}U. In the region of the dashed lines the resonances are too close together for the experimental data to be displayed on the scale of the figures. Note that both the horizontal and vertical scales are logarithmic. (Data from Garber, D. I. & Kinsey, R. R. (1976), *Neutron Cross Sections*, vol. II, Upton, New York: Brookhaven National Laboratory.)

Table 9.1. *Distribution of energy release on the induced fision of a nucleus of* ^{235}U

		MeV
Kinetic energy of fission fragments		167
Kinetic energy of fission neutrons		5
Energy of prompt γ-rays		6
Sub-total of 'immediate' energy		178
Electrons from subsequent β-decays	8	
γ-rays following β-decays	7	
Sub-total of 'delayed' energy		15
Neutrino energy		12
		205

FISSION:

Due to pairing considerations n + A → B + C + several n for thermal neutrons can occur only for odd-A nuclei. Examples are ^{233,235}U and ^{239,241}Pu.

The key fact for ^{235}U is that it occurs in nature (0.7% of natural U ore) and that it emits an average of 2.5 neutrons per fission, with about 0.02 neutrons delayed by about 13 seconds (evaporation from fission fragments) which allows easy control of the fission rate.

We get about 205 MeV of kinetic energy from each fission, with the kinetic energy carried by the fission fragments typically being around 170 MeV.

Reactors usually use fuel with a high "thermal" (1 eV or less) n fission total cross section, whereas for bombs and "fast" reactors a fuel is needed that has a high n fission total cross section for 2 MeV neutrons. Thus people speak of "slow" and "fast" fission processes.

Suppose we have a fuel that is a mixture of ^{235}U and ^{238}U, as found in nature.

Then the mean free path of a neutron is about $\ell = (\rho_{\text{nuc}}\bar{\sigma}_{\text{tot}})^{-1}$ where $\bar{\sigma}_{\text{tot}} = c\sigma_{\text{tot}}^{235} + (1-c)\sigma_{\text{tot}}^{238}$ and since the density is about 4.8×10^{28} nuclei per cubic meter and the average cross section is about 7 barns, the mean free path is around 3 cm, a distance which a 2 MeV n can travel in 10^{-9} sec.

In a bomb you would use pure ^{235}U produced by isotope separation, or pure ^{239}Pu, made by n-capture on ^{238}U in reactors.

The competition to fission is (a) escape of neutrons from the system, and (b) radiative capture. [Avoid resonances!]

Let q be the probability of fission, and ν be the average number of neutrons produced per fission. Then we get $(\nu q - 1)$ new neutrons, produced in time t_p of 10^{-8} sec.

Then $n(t + dt) = n(t) + (\nu q - 1)n(t)(dt/t_p)$ so that

$$n(t) = n(0)\exp((\nu q - 1)t/t_p).$$

For ^{235}U we get an exponential increase and an explosion if $(\nu q - 1) > 1$ or $q > (1/\nu) \simeq 0.4$.

Thus to build a bomb you need a solid piece of ^{235}U about 8.7 cm in radius with a corresponding mass of about 50 kg. The standard approach is to bring a large number of pieces explosively together to assemble this "critical mass," often with a small source of neutrons at the center. The numbers for ^{239}Pu are different.

Reactors:

The problem is to "cool" 2 MeV neutrons down to less than 1 eV without them being captured by ^{238}U.

Thermal reactors: Natural U (0.7% ^{235}U) in ceramic (oxide) form is placed in long, thin fuel rods (cans) so that to get from one rod to another neutrons must scatter repeatedly from a "moderator." The usual moderators in such reactors are graphite (dangerous!) and heavy water (D_2O).

If the fuel is enriched to 2 to 3% in ^{235}U, water can be used as a moderator.

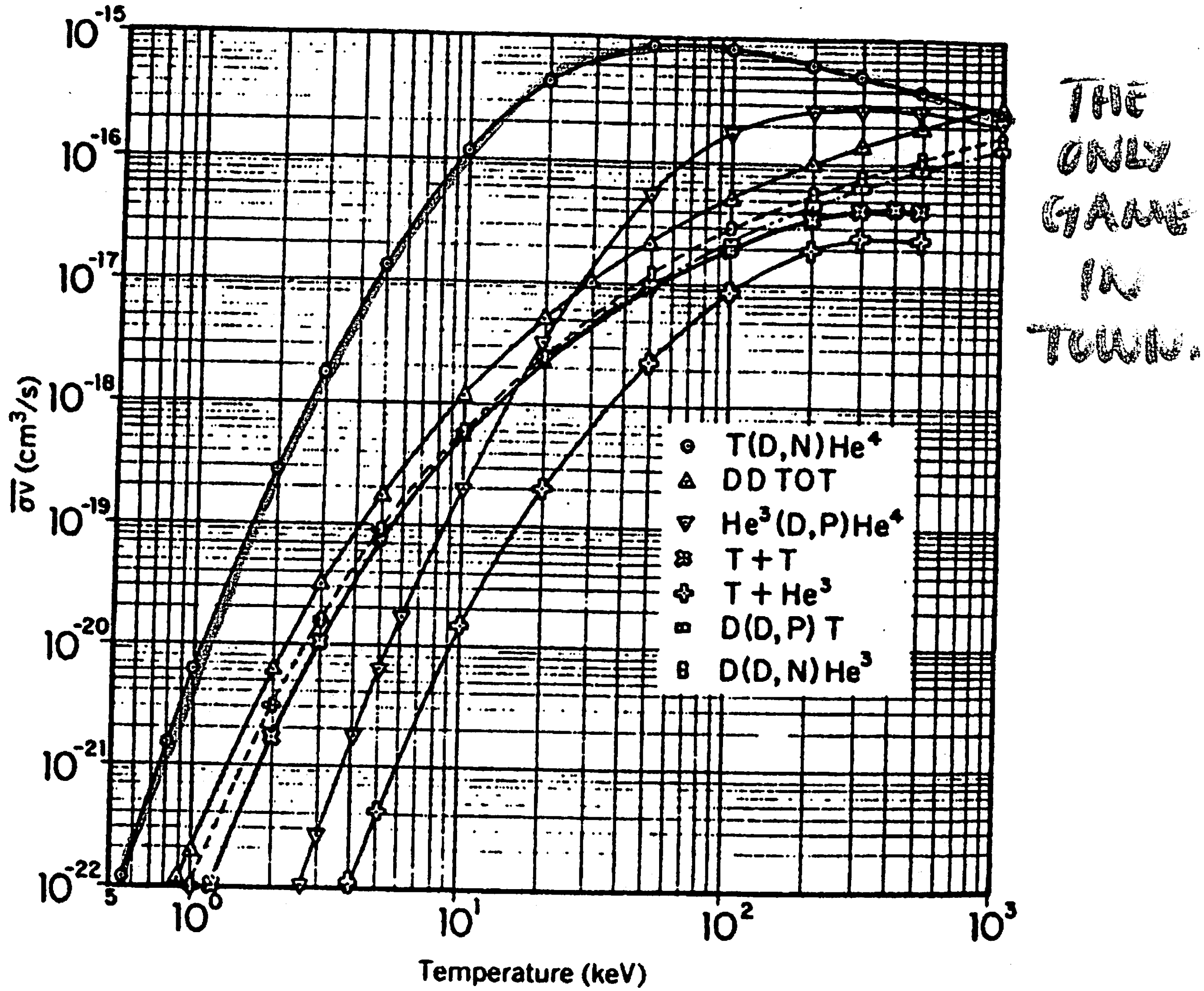
The standard reactors used in the USA were originally designed as power plants for nuclear submarines, and are either "boiling water" reactors or "pressurized water" reactors.

Fast Reactors: ^{239}Pu fissions with 2 MeV neutrons so no moderator is needed, and you get nearly 3 neutrons per fission. The coolant is generally a liquid metal... usually sodium. The operational reliability of such reactors has not been impressive, to say the least.

Steady State:

In a reactor you want $(\nu + \nu_d)q - 1 = 0$, which is achieved in the standard approach first used by Fermi, by mechanically moving control rods that strongly absorb neutrons in and out of the reactor.

Reactors are designed so that normally $\nu q - 1 < 1$, and then q is varied until a steady state is achieved. It is also vital to design the reactor so that $dq/dT < 0$, otherwise you are asking for big trouble. The infamous reactor at Chernobyl was not only made of graphite (!) but had $dq/dT > 0$ when run at low power!



ues of $\langle \sigma v \rangle$ averaged over a Maxwell–Boltzmann energy distribution for vari
s (Keefe 1982).

THERMAL FUSION:

Remember that the particle flux is ρv so that the reaction rate is the flux times the total cross section.

For two merging systems, in the center of momentum system, with μ the reduced mass,

$$P(v)dv = \left[\frac{2}{\pi}\right]^{1/2}\left(\frac{\mu}{k_BT}\right)^{3/2} \exp(-\mu v^2/(2k_BT))v^2dv.$$

The reaction rate per unit volume would then be

$$K\rho_a\rho_b\langle v\rho_{ab}\rangle,$$

where $K = 1$ if a and b are different, or $1/2$ if they are the same.

The average is over the Boltzmann distribution:

$$\langle v\sigma_{ab}\rangle = \int_0^\infty v\sigma_{ab}P(v)dv.$$

If the number of particles per unit volume is n then $R = (n^2/4)\langle\sigma v\rangle$.

The energy delivered in time τ if Q is the energy release per fusion is

$$E_f = \frac{n^2}{4}\tau Q\langle\sigma v\rangle.$$

The total KE of the hot plasma (ions and electrons) is $E_K = 3nk_BT$.

Thus we are not accomplishing anything unless $E_f > E_K$, which means

$$n\tau > \frac{12k_BT}{\langle\sigma v\rangle Q}.$$

This is the so-called **Lawson Criterion.**

Suppose we can make it to a temperature of 10 keV. Then we need $n\tau > 0.7 \times 10^{15}$ sec/cm^3.

The two main approaches to this goal have been:

(1) ***Magnetic Confinement*** n (plasma) about 10^{14} particles/cm^3, so τ must be more than 10 seconds!

(2) ***Inertial Confinement*** n (liquid) about 10^{25} particles/cm^3 so τ must be around 10^{-10} sec.

The problem with Magnetic Confinement has always been magnetohydronamic instabilities, which cause the plasma to break up into clumps in a relatively short time, much shorter than 10 seconds. A continuous plasma is needed for Joule heating by sending a current through it.

The problem with Inertial Confinement is that you have only 10^{-10} sec to heat the droplet to 10 keV.

The heat capacity of a 1 mm drop of d-t liquid is about 10^{-4} keV/J, so the power needed to be delivered to get the drop up to 10 keV is about 10^{15} Watts. Instabilities of compression tend to doom this effort.

CONCEPTUAL PROBLEMS:

(1) 80% of the released KE is carried by neutrons. How do you boil water with neutrons?

(2) What kind of power plant is it that *needs to be plugged into an existing electrical grid to work???*

Convenient Astrophysical Parameterizations:

$$\sigma(E) = \frac{1}{E} S(E) \exp[-\sqrt{E_G/E}],$$

where

$$E_G = 2\mu c^2 \left(\frac{\pi k_e Z_1 Z_2 e^2}{\hbar c} \right),$$

and to a good approximation $S(E) \simeq S(0)$.

The relevant energies for astrophysics are ≤ 1 keV.

Based on theories of the weak interaction,

$$S_{pp} \simeq 3.88 \times 10^{-25} \text{ MeV barns},$$

Measured values for relevant nuclear cross sections are

$$S_{pd} \simeq 2.5 \times 10^{-7} \text{ MeV barns},$$

and

$$S_{hh} \simeq 4.7 \text{ MeV barns}.$$

Instead of $p + p \rightarrow p + n + e^+ + \nu_e \rightarrow d + e^+ + \nu_e$, about 1 time in 400 the process proceeds by electron capture from the dense plasma:

$p + e^- \rightarrow n + \nu_e$ followed by $p + n \rightarrow d + \gamma$.

Why is S_{pp} so small?

Consider that the probability is approximately given by the ratio of the nuclear time scale to the weak time scale. For a nuclear process, the time scale is 10^{-23} sec, while a typical fundamental weak process is neutron decay, which takes 887 seconds. Thus the ratio is about

$$1.1 \times 10^{-26}.$$

Now the pp cross section is around 36 b at 1 MeV so we would estimate the weak cross section for p + p resulting in d to be around

$$\sigma_{\text{make d}} \simeq (36\text{ b})(1.1 \times 10^{-26}) \simeq 4 \times 10^{-25}\text{ b},$$

which agrees well with more elaborate calculations based on accurate theories of weak processes.

Our sun contains about 10^{57} protons, of which 34% are in the core.

The reaction rate is about 3.6 $\times 10^{38}$ protons per second.

The time to consume half of the core protons is then $[(0.34)(0.5)\ 10^{57}]/(3.6 \times 10^{38}) \simeq 5 \times 10^{17}$ sec, which is about 10^{10} years.

Since the sun is about 4.6×10^9 years old, it is roughly halfway through its main-sequence lifetime.

Power Density:

For a human, M is around 75 kg, luminosity about 75 Watts (infrared) so $\mathcal{L}/M$ is 1 Watt/kg.

For the sun, $\mathcal{L}_\odot/M_\odot$ is (4×10^{26} Watts)/(2×10^{30} kg) $\simeq 2 \times 10^{-4}$ Watts/kg, about the same as a compost heap!

THE MAIN SEQUENCE:

$$\mathcal{L} = 4\pi\sigma R^2 T^4.$$

$$\mathcal{L} \propto M^{3.5}.$$

$$\tau_{\rm MS} \simeq 10^{10}\ {\rm yr}\left(\frac{M}{M_\odot}\right)\left(\frac{\mathcal{L}_\odot}{\mathcal{L}}\right)$$

~~which reduces to~~

$$\tau_{\rm MS} \simeq 10^{10}\ {\rm yr}\left(\frac{M}{M_\odot}\right)^{-2.5}.$$

Ranges:

$$0.1 < R/R_\odot < 20,$$

$$0.1 < M/M_\odot < 100,$$

$$10^{-4} < \mathcal{L}/\mathcal{L}_\odot < 10^6,$$

$$2000K < T < 40,000K.$$

No stars are seen with $M \geq 170 M_\odot$ or $M < 0.08 M_\odot$.