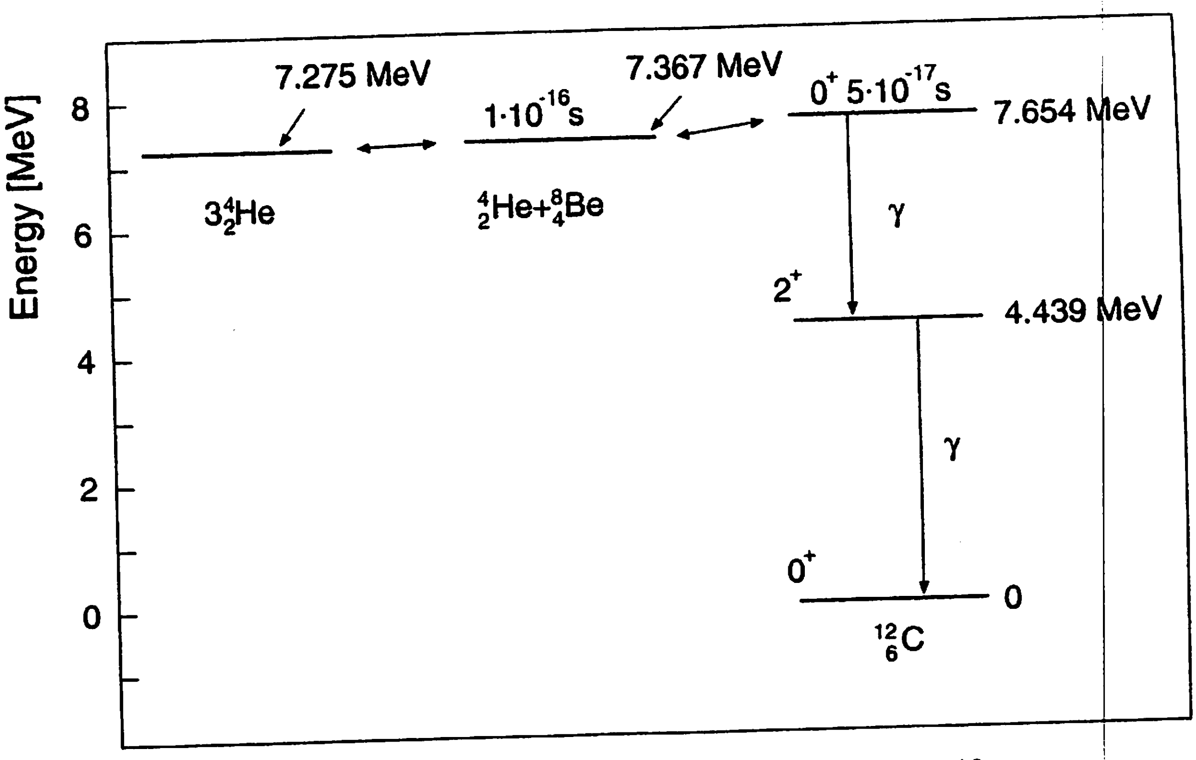


Fig. 19.11. Energy levels of the system: $3\,\alpha$, $\alpha + {}^8$Be and ^{12}C. Just above the ground states of the $3\,\alpha$ system and of the $\alpha + {}^8$Be system there is a 0^+ state in the ^{12}C nucleus, which can be created through resonant fusion of ^{4}He nuclei. This excited state decays with a 0.04 % probability into the ^{12}C ground state.

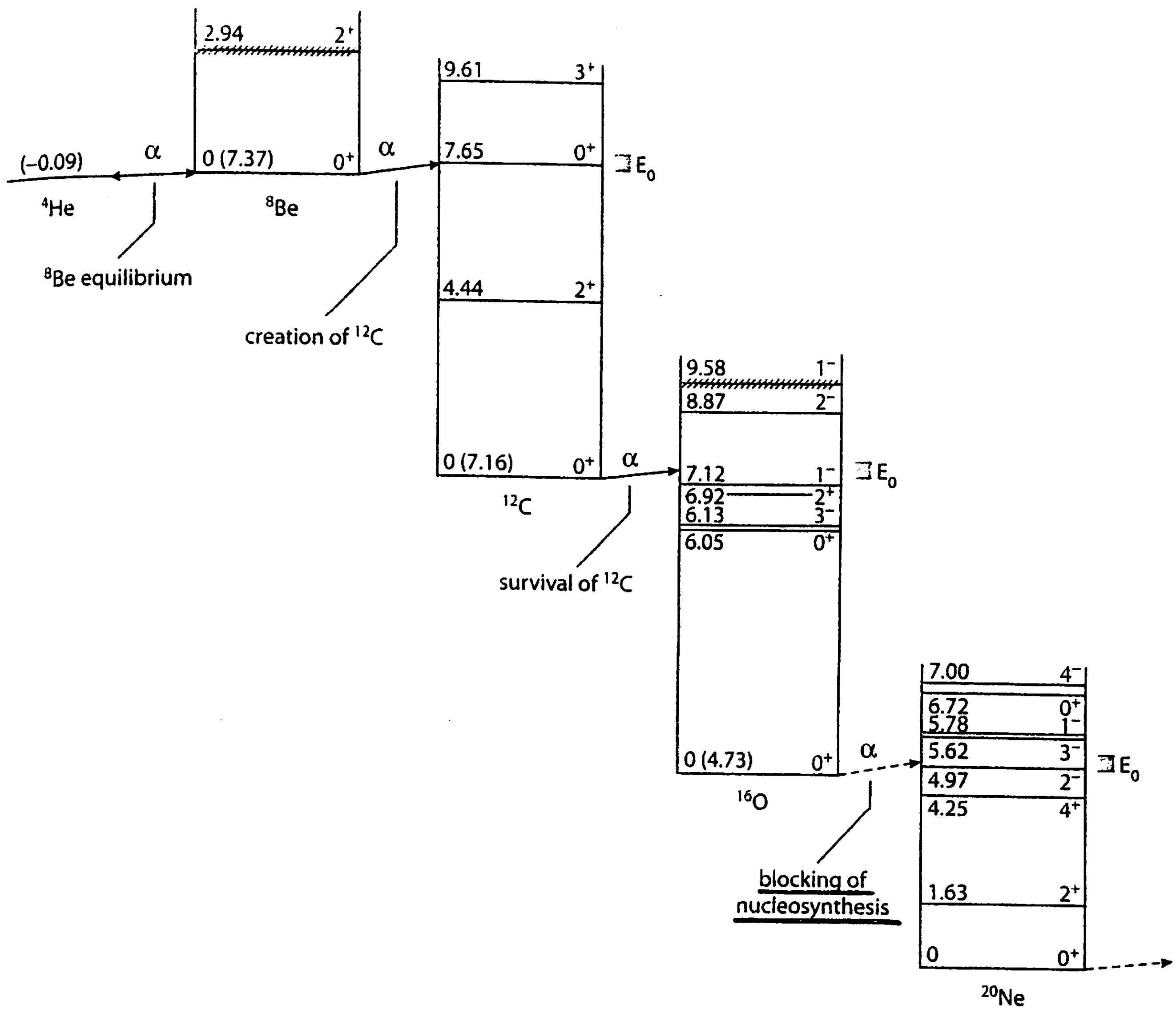


Figure 12.11 Energy levels of nuclei participating in thermonuclear reactions during the helium burning stage in red giant stars (adapted from [RR88]). The survival of both ^{12}C and ^{16}O in red giants, believed to be the source of terrestrial abundances depends upon fortuitous circumstances of nuclear level structures and other properties in these nuclei.

Final Stages of Stellar Evolution:

• $M < 1.4 M_\odot$, White Dwarf supported by electron Fermi pressure... the star slowly goes out... to black dwarf. If matter is dumped onto a White Dwarf from a companion star, when it reaches the mass limit that electron pressure can no longer support, it will undergo a type 1a supernova, synthesizing heavy elements beyond iron. Such supernovae are vitally important in establishing a cosmic distance scale, because the exploding star has a specific *known mass*, and thus its absolute luminosity, and light-curve, are always the same.

• $1.44 M_\odot < M < 3 M_\odot$, "neutron" star. Actual composition not well known. Collisions of binary neutron stars turn out to be vitally important in producing a large range of nuclei beyond A = 43, and are the topic of intense current research, both observational and theoretical.

• $M > 10 M_\odot$, black hole. Black holes play no further role in nucleosynthesis. Colliding black holes are the easiest-to-spot sources of gravitational radiation, with colliding neutron stars next in line in terms of intensity.

What lies in between these limits? Any guesses?

Neutron stars and black holes and some unknown states are the results of various types of supernovae, particularly type 2. The processes that occur during supernovae break down already-formed nuclei in the core, and make new nuclei by the so-called r-process. Copious amounts of neutrinos are produced, but the only supernova so far observed by existing neutrino telescopes was SN1987, which occurred in the LMC. Calculations of supernova explosions were one of the earliest physics applications of nuclear weapons programs developed during the 1960s and 1970s. In addition to producing heavy elements, supernovae are vital in generating shock waves that trigger formation of new stars on a massive scale. There is chemical evidence in our own solar system of the supernova that triggered its formation.

Neutron stars are thought to have a crust of semi-solid metal, perhaps mainly iron. Just inside is a dense gas of nuclei with a large neutron excess, then a Fermi liquid of protons and neutrons... inside that, is anyone's guess. The core is a mystery, due to lack of knowledge of the nuclear equation of state.

Are there stars that are mainly a quark-gluon plasma. so-called "quark stars"? How could you tell?

Vacuum Energy, Dark Energy?

Remember that quantum field theory contains an inherent infinity, due to the vacuum energy

$$H|0\rangle = \int_{-\infty}^{\infty} d^3k \frac{\hbar\omega_k}{2}.$$

The standard solution is to observe that the Standard Model does not include gravity, so that the integral should be cut off at the Planck Mass,

$$m_p = \sqrt{\frac{\hbar c}{G}}.$$

This is about 1.22 $\times 10^{19}$ GeV. The corresponding energy density of the vacuum works out to be $\rho \simeq 10^{93}$ gm/cm^3.

There is such a vacuum energy in Einstein's theory of gravity, the so-called Cosmological Constant, aka Dark Energy. But based on observations of distant type 1a supernovae, this energy has to be less than about

$$10^{-29} \text{ gm/cm}^3.$$

Note the discrepancy of about 122 orders of magnitude!!!!

An Equation for the Universe:

Choose a metric like $ds^2 = c^2dt^2 - a^2(t)d\mathbf{r}^2$. The dimensionless function $a(t)$ incorporates all the dynamics of a particular universe.

With this metric, the Einstein field equations for gravity, applied to the universe, give

$$\left(\frac{\dot{a}}{a}\right) = \left[\frac{(8\pi G)}{3}\right]\rho - \frac{k}{a^2}.$$

Here ρ is the density and $k = 0$ for flat space, or $k = \pm 1$ for positive or negative overall curvature.

Also we get

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p),$$

where p is the pressure corresponding to ρ.

The equation of state is usually written $p = w\rho$. Note that when the pressure is positive, $w > 0$, we get a universe with slowing expansion, $\ddot{a} < 0$.

Einstein's field equations also lead to a continuity equation of the form

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0.$$

Here H is the so-called Hubble parameter, essentially the same as $(\dot{a}/a)$.

Einstein assumed the universe is infinitely old and static, since physical constants show no time dependence. In addition to the matter density $\rho_M = k/(4\pi G a^2)$, he introduced a "cosmological" term $\rho_\Lambda = (\rho_M)/2$, a choice which results in $\ddot{a} = \dot{a} = 0$, and a closed, finite universe, $k = +1$.

But almost at once it was discovered the universe is expanding! And as time has gone on, we find that the universe is accelerating in its expansion, $\ddot{a} > 0$, and that as far as experiment is concerned, $k = 0$, very precisely... the universe is asymptotically flat.

Critical Density:

For a $k = 0$ universe,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left[\frac{8\pi G}{3}\right]\rho.$$

If we define the usual Hubble constant H_0 by $v = H_0 D$, then we have a critical density

$$\rho_c = \frac{3H_0^2}{8\pi G}.$$

It is customary to define

$$\Omega = \frac{\rho}{\rho_c}.$$

Based on experimental observations, especially of distant type Ia supernovae, $H_0 = 68$ km/sec per megaparsec, and $\Omega = 1$.

It is also found that a cosmological term as invented by Einstein actually exists. Only 0.3 of the required Ω is due to matter. The remaining 0.7 comes from vacuum energy. As empty space expands, the amount of empty space increases so the vacuum energy increases. There is also a very readily observable effect on the wavelength of light... as space expands, wavelengths increase in transit.

Just from the 2nd Law,

$$ma = \frac{mv^2}{r} = F(r),$$

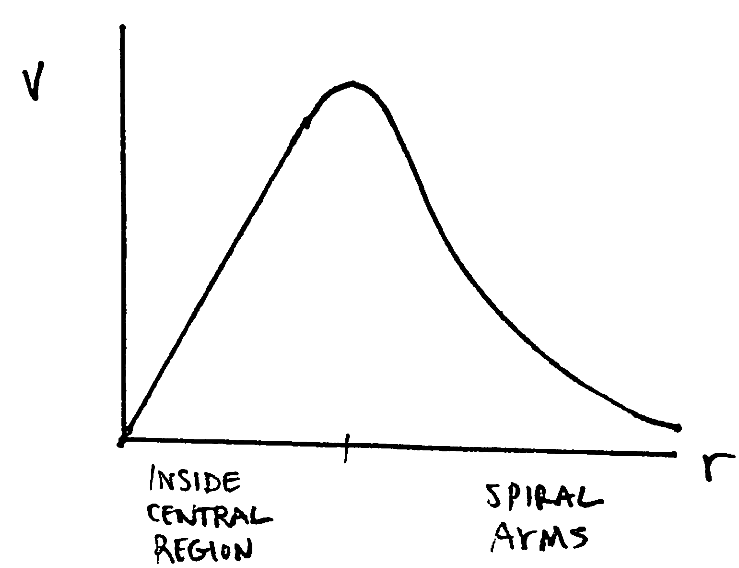
we know that if we were orbiting outside a compact mass, like the sun, or the center of the galaxy, we would find

$$v \propto 1/\sqrt{r}.$$

On the other hand, if we were orbiting inside a uniform distribution of mass, roughly like the center of the galaxy, we would, since $F(r) \propto r$, expect that

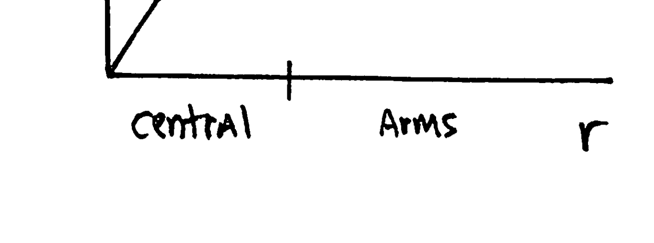
$$v \propto r$$

Thus, for stars in the arms of any galaxy we would expect



The distribution we actually see, $v \propto$ constant, suggests that usually the entire galaxy being studied is within a roughly spherical distribution of matter, with a diameter about 10 - 20 times the galactic diameter, and with a density decreasing as we move from the center of the distribution to its fringe.

This behavior is seen for almost all galaxies that we can observe. Every once in a while we see a galaxy that seems to have no dark matter "halo," and there are a very few suggestions of galaxies that are almost entirely dark matter, with very sparse population of stars.



GETTING THE COMPOSITION OF THE UNIVERSE:

The last scattering surface is of course the dark night sky, surrounding us, subtending a 4π solid angle. The observed temperature variations are $\Delta T(\theta, \phi)$. Physicists want to expand any function of polar angles in terms of $Y_{\ell,m}(\theta, \phi)$ since these spherical harmonics form a complete, orthogonal set of states.

So

$$\frac{\Delta T}{T_0} = \sum_{\ell m} \alpha_{\ell m} Y_{\ell m}(\theta, \phi)$$

and we can find the coefficients by

$$\alpha_{\ell m} = \int Y^*_{\ell m}(\Delta T / T_0) d\Omega.$$

The sum starts at $\ell = 1$.

If we assume random Gaussian fluctuations and average appropriately we can define C_ℓ by

$$\langle (\Delta T / T_0)^2 \rangle = \sum_\ell \frac{2\ell + 1}{4\pi} C_\ell,$$

since $\sum_m |Y_{\ell m}|^2 = (2\ell + 1)/(4\pi)$.

There are technical considerations resulting from the fact that the observed $\widehat{C_\ell}$ in the space we can observe is different from the true C_ℓ, but an analysis leads to results like

$$\langle(\widehat{C}_\ell - C_\ell)^2\rangle = (2/(2\ell+1))C_\ell^2.$$

Since as you remember from 373, the Y's are like standing waves of particular and specific wavelength, there is a correspondence between ℓ and θ, such that

$$\theta_\lambda = \frac{2\pi}{\ell}$$

so that

$$\theta_{\rm res} \simeq \frac{\pi}{\ell}.$$

WMAP could resolve to $\ell \simeq 780$, while PLANCK could resolve to $\ell \simeq 2160$. It would not be difficult to construct orbital microwave telescopes which could resolve to much larger maximum ℓ.