Types of nuclear instability:

- $\alpha$ Decay—Heavy nuclei emit $^4\text{He}$ nuclei via barrier penetration.
- $\beta$ Decay—Nuclei with too many $n$ or $p$ are unstable to weak decay of $p$ to $n$ or $n$ to $p$. In this decay electrons and positrons are emitted.
- $\gamma$ Decay—Following either $\alpha$ or $\beta$ decay, the final nucleus is almost always left in an excited state, and will de-excite by emitting a photon. But this photon has a million times the KE of a photon of visible light, because the spacing between nuclear energy levels is a million times the spacing of atomic and molecular energy levels!

In all quantum systems, the probability of any transition is independent of time. This means that if a system has a 50% probability of making a certain transition in 1 hour, and it has been sitting in its initial state for a year, it still has a 50% probability of making the transition in the next hour!
To express this mathematically,

\[ \frac{1}{N} \frac{\Delta N}{\Delta t} = -\lambda = -\frac{1}{\tau}, \]

where \( \lambda \) is the transition probability per unit time, and \( \tau \) is the average lifetime.

Therefore

\[ \frac{\Delta N}{\Delta t} = -\frac{N(t)}{\tau}, \]

which has the solution

\[ N(t) = N(0) \exp[-\lambda t] = N(0) \exp(-t/\tau). \]

It is customary to define the half-life \( T_{1/2} \) as the time during which the decay has a 50\% probability of happening. Then, without calculus, you can instantly see that

\[ N(t) = N(0) \left[ \frac{1}{2} \right]^{t/T_{1/2}}. \]

An ancient unit is 1 Cu = 3.7 \times 10^{10} per second. The modern unit is 1 Bq = 1 per second.

Note that the rate of decay at any time, \( R(t) \), is given instantly by

\[ \frac{N(t)}{\tau}. \]
NUCLEAR PROCESSES:

\[ A + a \rightarrow b + B \]

\[
(M_A + M_a)c^2 + KE_i = (M_B + M_b)c^2 + KE_f.
\]

We define the \( Q \) value for the process as

\[ Q = KE_f - KE_i. \]

Therefore

\[ Q = [M_A + M_a - M_B - M_b]c^2. \]

If \( Q \) happens to be negative then there is a minimum kinetic energy in the "lab frame of reference" (\( A \) at rest) for which the process can occur:

\[ KE_{a,\text{min}} = \left[ 1 + \frac{M_a}{M_A} \right]|Q|. \]
A radioactive nucleus has an average lifetime of only 100 minutes. If you started with a million such nuclei, how many would be left after 100 min? What is the half-life of this nucleus?

Answer: the number remaining would be $3.7 \times 10^5$ after 100 min. The half-life would be 69.3 min.

In a nuclear reaction, $a + A \rightarrow B + b$, the masses of initial and final states are 14070 and 14080 MeV respectively. What is the minimum lab kinetic energy of $a$ that would allow this reaction to take place? Assume $M_a/M_A$ is 0.1.

Answer: 11 MeV.

A nuclear process that is important astrophysically (in red giant stars) is the following. What goes in the blank?

$$^4\text{He} + ^{12}\text{C} \rightarrow \underline{\text{_______}} + \gamma.$$