Batteries have internal resistance, so we write:

\[ \Delta V - IR = \mathcal{E} - Ir - IR = 0. \]

Resistances in series:

\[ R_{\text{tot}} = \sum_i R_i. \]

Resistances in parallel:

\[ \frac{1}{R_{\text{tot}}} = \sum_i \frac{1}{R_i}. \]

Networks of resistors: Identify groups, combine, and continue to combine until you are left with one equivalent resistor.

**Kirchhoff’s Circuit Rules:**

1. At any circuit junction, \((\sum_i I_i)_{\text{in}} = (\sum_i I_i)_{\text{out}}.\) Charge is conserved.

2. Around any circuit loop, \((\sum_i \Delta V_i)_{\text{rises}} = (\sum_i \Delta V_i)_{\text{drops}}.\) Energy is conserved.
Kirchhoff’s Rules:

(1) The algebraic sum of currents at a junction is always zero. That is, the sum of currents entering the junction equals the sum of currents leaving the junction. **Reason:** Charge is conserved!

(2) The algebraic sum of the changes in potential around any closed conducting loop must equal zero. That is, the sum of potential rises equals the sum of potential drops, around the loop. **Reason:** $\oint \mathbf{E} \cdot d\mathbf{r} = 0$, the electric field $\mathbf{E}$ in this situation is conservative.
Example of Kirchhoff Law Application:

$\mathcal{E}_1 = 7 \text{ V}$, $\mathcal{E}_2 = 5 \text{ V}$, and $R_i = (i + 1) \Omega$.

Loop with $R_1$ and $R_2$:

$I_1 = I_2 + I_3$, Rule 1

$\mathcal{E}_1 - I_1 R_1 - I_2 R_2 = 0$, Rule 2.

Loop with $R_2$ and $R_3$:

$\mathcal{E}_2 - I_2 R_2 + I_3 R_3 = 0$.

Eliminate $I_1$ and solve for $I_2$ in terms of $I_3$, using the second loop equation:

$I_2 = \frac{\mathcal{E}_2 + I_3 R_3}{R_2}$.

Now solve directly for $I_2$ using the first loop equation, to get eventually:

$I_2 = \frac{\mathcal{E}_1 - I_3 R_1}{R_2 + R_1}$. 
Do the algebra, combining the two equations to solve for $I_3$:

$$I_3 = \frac{\mathcal{E}_1 R_2 - \mathcal{E}_2 (R_1 + R_2)}{R_3 (R_1 + R_2) + R_1 R_2}.$$

Plugging in the numbers finally yields $I_3 = -0.154$ A.

Now solve directly for $I_2$ using the equation for $I_2$ involving only $I_3$: Result: $I_2 = 1.46$ A. So now use the very first equation, to get $I_1 = 1.31$ A.

Now you can check directly by plugging back into the two original loop equations.
Guess a current direction. Then, look at the various loops in the circuit.

- if you loop with the current, then $\Delta V < 0$ across any resistor.
- If you loop against the current, then $\Delta V > 0$ across any resistor.
- If you loop with the current, then $\Delta V = \mathcal{E}$ across any battery.
- If you loop against the current, then $\Delta V = -\mathcal{E}$ across any battery.

**RC Circuit:**

Charging: $q(t) = Q[1 - \exp(-t/RC)]$ where $Q = C\mathcal{E}$.

Discharging: $q(t) = Q \exp(-t/RC)$ where $Q = C\mathcal{E}$. 
Circuit with Capacitor and Resistor:

Charging,

\[ \mathcal{E} - IR - \frac{q}{C} = 0 \text{ so } \mathcal{E} = R \frac{dq}{dt} + \frac{q}{C}. \]

Thus

\[ \frac{dq}{q - \mathcal{E}C} = - \frac{dt}{RC}. \]

If \( q = 0 \) at \( t = 0 \) then

\[ \ln[1 - \frac{q}{\mathcal{E}C}] = - \frac{t}{RC}, \]

or finally

\[ q(t) = \mathcal{E}C \left[ 1 - \exp(-t/RC) \right]. \]

Of course

\[ I = \frac{dq}{dt} = \frac{\mathcal{E}}{R} \exp(-t/RC). \]
Charging capacitor in RC circuit:

\[ V_C = \mathcal{E} \left[ 1 - \exp(-t/RC) \right] \text{ and } V_R = \mathcal{E} \exp(-t/RC). \]

Discharging capacitor across R:

\[ V_C = \frac{Q}{C} \exp(-t/RC), \quad I(t) = -\frac{Q}{RC} \exp(-t/RC) \]

and \[ V_R = -\frac{Q}{C} \exp(-t/RC). \]
Discharging RC circuit:

\[ \mathcal{E} = 0 \text{ so } -IR - \frac{q}{C} = 0, \]

so

\[ \frac{dq}{q} = -\frac{1}{RC}dt, \]

and

\[ q(t) = q_0 \exp[-t/RC]. \]

\[ I(t) = -\frac{q_0}{RC} \exp[-t/RC]. \]
**RC Circuit:**

Charging:

\[ \varepsilon - IR - q/C = 0 \]

Solving for \( q(t) \) we find

\[ q(t) = \varepsilon C \left[ 1 - \exp(-t/RC) \right], \]

and

\[ I(t) = \frac{\varepsilon}{R} \exp(-t/RC). \]

Discharging:

\[ q(t) = q_0 \exp(-t/RC). \]

\[ I(t) = -\frac{q_0}{RC} \exp(-t/RC). \]