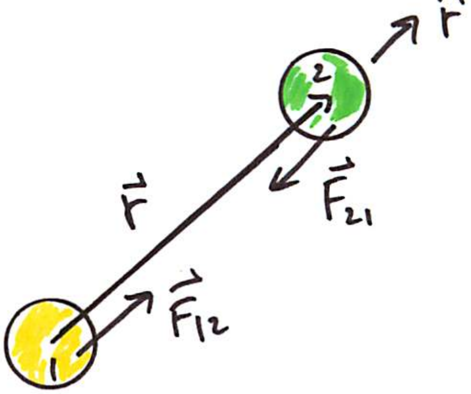
Newton's Law of Gravity:

$$\mathbf{F}_{21} = -\widehat{\mathbf{r}}\frac{GM_1M_2}{r^2}.$$



Gravitational acceleration on a planetary surface:

$$g_p = \frac{GM_p}{R_p^2}.$$

Circular orbits:

$$v = \sqrt{\frac{GM}{r}} \propto 1/\sqrt{r}.$$

Gravitational potential energy:

$$\mathrm{PE}_g = -\frac{GMm}{r}.$$

In another solar system is a planet, Ranar, with 100 times the mass of the earth and 10 times the radius. What is the gravitational acceleration on the surface of Ranar? Answer: 10 m/s^2.

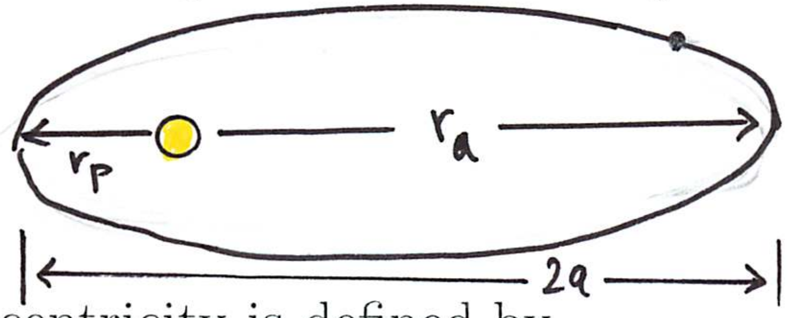
Show that a satellite in a circular orbit of radius r around the earth has speed $v = R_e\sqrt{g/r}$ where R_e is the earth's radius.

An object is held at rest a distance $r = 10R_e$ from the center of the earth, and dropped. Neglecting the rotation of the earth on its axis, and atmospheric friction, show that the speed at which it hits the surface is $v = \sqrt{1.8gR_e}$.

Show that a satellite orbiting earth in a circular orbit just beyond the atmosphere (which extends upward about 20 miles) has a speed of 8 km/s (about 5 mi/s) and has an orbital period of about 85 minutes.

Elliptical Orbit:

The important parameters for an elliptical orbit are the *eccentricity* e and the *semimajor axis* a.



The eccentricity is defined by

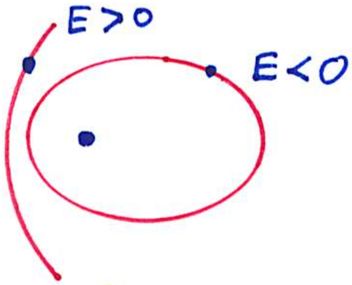
$$e = \frac{r_a - r_p}{r_a + r_p}.$$

The orbital period only depends on a... T is proportional to $a^{3/2}$.

$$T = 2\pi\sqrt{\frac{a^3}{GM}}.$$

$$v = \sqrt{GM\left[\frac{2}{r} - \frac{1}{a}\right]}$$

- BOUND ORBIT: $E < 0$.
- UNBOUND OBJECT: $E > 0$.
- Escape speed: $v_e = \sqrt{2GM/r}$.



Kepler's Rules of Planetary Motion:

(1) The planets and moons orbit in ellipses. For planets the sun is at one focus, for moons the planet orbited is at one focus.

The only force $\mathbf{F}$ which results in such orbits is $\mathbf{F} = -\widehat{\mathbf{r}}C/r^2$, where C is a constant.

(2) The radius vector from sun to planet sweeps out equal areas in equal times.

This is a consequence of conservation of angular momentum since $\mathbf{F} \propto -\widehat{\mathbf{r}}$.

(3) $T^2 \propto a^3$ for any orbiting body.

This is a consequence of Newton's 2nd Law combined with Newton's Law of Gravity.

The orbit of a planet around the sun would be an ellipse if that planet were alone in space. But in our real solar system there are many planets and the larger planets have many moons. Thus the force of gravity on any given planet or moon varies in a way depending to some extent on every other body in the solar system. It came as a great shock to astronomers and physicists about a century ago when they realized there is NO STABLE SOLUTION for planetary orbits under these conditions. Right now things seem to be orderly and regular, but in the past things were completely chaotic, and they will eventually become chaotic again.

The most recent chaotic era in the inner solar system, the so-called Late Heavy Bombardment, occurred 4.1 to 3.8 billion years ago. It is remarkable that life on earth appears to have originated right at the end of this era! [Slightly earlier than 3.7 billion years ago.] Unpredictable instabilities in the present state of our solar system occur over time scales of about 100 million years. In other words, it is impossible to predict the orbit of the earth or of any other planet 100 million years in the future.