• A neutron has a radius of about 0.83 fm, while another particle called the pion has a radius of about 0.66 fm. The two particles can interact via the strong interaction, which has an effective range of only about 1 to 3 fm. Suppose we do an experiment where pions are elastically scattered from neutrons. What is a good estimate of the total cross section for this process?

Solution: The probability distributions for the internal states of the particles cannot overlap, because they are both made of real quarks (and also antiquarks for the pion), which are fermions, occupying some of the same states. We will learn later that the Pauli Principle requires that identical fermion particles in the same state cannot occupy the same region of space. So the probability distribution of the pion cannot overlap the probability distribution of the neutron. So we could estimate the total cross section to be $\sigma = \pi(r_n^2 + r_p^2)$. This gives 3.14(0.69 + 0.44) fm^2 = 3.6 fm^2. The usual unit of total cross sections is the mb, and this is 36 mb, because 1 square fermi is 10 mb. As a cruder estimate we could approximate the range of the strong interaction as 1.4 fm, a standard approximation, and then we would get 3.14(2) = 6.3 fm^2, about twice the previous estimate, but both of these are order-of-magnitude estimates, and they do agree within an order of magnitude, in fact within roughly a factor of 2.

• Quantum physics was unknown when Rutherford did his experiments, so he analyzed the results with classical Newtonian physics. His calculations agreed perfectly with his data, assuming the projectiles and target were point charges. Why did his experiment not provide information about the radius of the nucleus, but only its existence?

Solution: In order to detect the size and shape of the nucleus, the probability wavelength of the incident beam would have to be comparable to the size of the nucleus. Any nucleus has a radius of $R = r_0 A^{1/3}$ and gold contains $A = 197$ protons and neutrons. So we get $R = 7$ fm. The beam of α particles would then need to have a momentum of at least $p = h/(7 \text{ fm})$. To get proper units for p we need proper units for hc, namely 1240 MeV-fm. That would give us $pc = (1240)/7 = 177$ MeV. The kinetic energy of the alphas, since we are still in the nonrelativistic regime— the mass of an alpha is a bit less than 4 times 940 MeV or less than 3760 MeV— we use the kinetic energy as $p^2/(2m)$ or more usefully as $(pc)^2/(2mc^2)$, so we need a beam energy for the alpha particles to be at least (177 MeV)^2/(2 times

3760 MeV) = about 7520 MeV. But the alphas in Rutherford's experiment came from the radioactive decay of Polonium, and these alphas all had an energy of around 5 MeV. So it would have been physically impossible for Rutherford's experiments to give, or depend upon, any information about the size of the atomic nucleus.

• Can you get the Coulomb law from the uncertainty relations applied to coupling of charge to the quantized electromagnetic field? Assume the absolute probability of a virtual photon being created near a charge Q and then vanishing near another charge Q is given by $\mathcal{P} = (k_e Q^2)/(\hbar c)$. Here k_e is the Coulomb constant since we expressed $\mathcal{P}$ in mks units... units not used in actual physics. P is a famous dimensionless number equal to about 1/137. For historical reasons it is known as the "fine structure constant," and usually written as α.

Solution: The resulting force should be $F = \mathcal{P} dp/dt$. Note that $\Delta E \Delta t \simeq \hbar$, and $\Delta p \Delta r \simeq \hbar$. For a photon, virtual or real, the total energy is pc and it travels at the speed of light. So approximate $r \simeq c\Delta t$ and $\Delta E \simeq \Delta pc$. Putting everything together we get $\mathcal{P} \Delta p/\Delta t = k_e Q^2/r^2$, Coulomb's law!