Chapter 11

Calculating the cross product using unit vectors:
$\widehat{\mathbf{i}} \times \widehat{\mathbf{j}} = \widehat{\mathbf{k}}$, $\widehat{\mathbf{j}} \times \widehat{\mathbf{k}} = \widehat{\mathbf{i}}$, etc.

Angular Momentum:

Of a particle: $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathrm{m}(\mathbf{r} \times \mathbf{v})$.

Of a rigid body: $\mathbf{L} = \mathbb{I}\vec{\omega}$.

2nd Law for Torques:

$$\Sigma_i \vec{\tau}_i = \frac{d\mathbf{L}}{dt}.$$

Conservation of Angular Momentum:

When no external torques act on a system, its total angular momentm **L** is a constant vector.

Bikewheel Angular Momentum:

Let $\mathbf{L}_w$ be the angular momentum of the spinning wheel, and $\mathbf{L}_{smb}$ be the angular momentum of the system of stool, man and (non-spinning) held wheel. Note that while $\mathbf{L}_w$ can point in any direction, $\mathbf{L}_{smb}$ can only point straight up or straight down, along the axis of the stool.

(1) Everything is at rest and the man begins to spin the wheel with his hand. Result: $\mathbf{L}_w + \mathbf{L}_{smb} = 0$, or $\mathbf{L}_{smb} = -\mathbf{L}_w$.

(2) The wheel is spinning, everything else is at rest, and the man stops the wheel's spin with his hand. $\mathbf{L}_w = \mathbf{L}_{smb}$.

(3) The spinning wheel is held horizontally, and then suddenly turned vertical: $0 = \mathbf{L}_{smb} + \mathbf{L}_w$.

(4) The spinning wheel is held vertically and then turned upside down: $\mathbf{L}_w = \mathbf{L}_{smb} - \mathbf{L}_w$.

EQUILIBRIUM!

A system is in equilibrium if $\mathbf{a}_{\rm cm} = 0$ and $\vec{\alpha} = 0$ about any axis.

In that case:

$$\sum_i \mathbf{F}_i^{\rm ext} = 0, \quad \text{and} \quad \sum_i \vec{\tau}_i^{\rm ext} = 0 \ \underline{\text{about any axis.}}$$