Kinematics of rigid body rotation:

$$\omega(t) = \omega(0) + \alpha t.$$

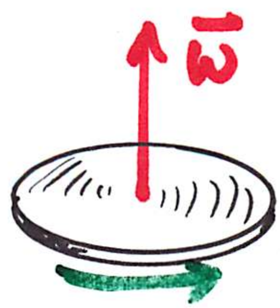
$$\theta(t) = \theta(0) + \omega(0)t + \frac{1}{2}\alpha t^2.$$

$$\omega^2 = \omega(0)^2 + 2\alpha[\theta - \theta(0)].$$

These expressions assume that the angular acceleration α is a **constant.**

Note that $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$ are **vectors,** with both magnitude and direction.

The magnitudes are given (for rigid body rotation) by $\omega = \Delta\theta/\Delta t$, and by $\alpha = \Delta\omega/\Delta t$, as $\Delta t \to 0$

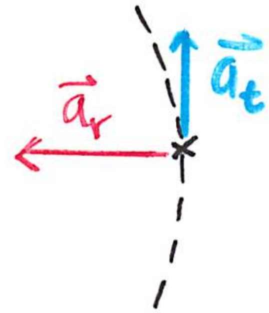


Relation between Angular and Linear quantities:

$$v_t = r\omega, \text{ and } a_t = r\alpha.$$

$$a_r = \frac{v_t^2}{r} = r\omega^2.$$

How to get the acceleration a of any point on a spinning rigid body:



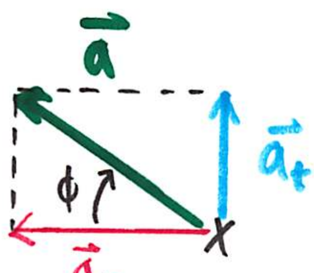
$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t.$$

$$a_r = r\omega^2 \text{ and } a_t = r\alpha.$$

Therefore $a = r\sqrt{\omega^4 + \alpha^2}$.

If we define ϕ as the angle between $\mathbf{a}$ and $\mathbf{a}_r$, then we instantly see that

$$\phi = \tan^{-1}\left[\frac{a_t}{a_r}\right].$$



A rigid disc spinning about its center has $\alpha = -2$ rad/sec^2 and $\omega(0) = 20$ rad/sec. How far does it turn in the next 10 seconds? [100 rad] What is its angular speed after 10 seconds? [Zero.] What is its angular speed after 15 seconds? [-10 rad/sec]

A spinning rigid body has ω of 10 rad/sec, α of 80 rad/sec^2, at a certain instant, and we fix our attention on a point 1 m from the axis of spin. What is the acceleration **a** of this point on the body, and what angle does it make with a line running from the point to the center of rotation? [The magnitude of **a** is 128 m/s^2 and the angle it makes is 38.7° with a radius line.]

A car is rounding a curve with a radius of 200 m at a speed of 30 m/s. The mass of the car is 1000 kg. If the car is on the verge of skidding, what is the coefficient of static friction μ_s between tires and road? [0.45]

• Show that for a conical pendulum, with string of length ℓ making an angle θ with the vertical, thus moving in a circle of radius $r = \ell \sin\theta$, the tension in the string is $T = mg/\cos\theta$ and the constant speed of the pendulum is

$$v = \sqrt{gr\tan\theta}.$$

Note the answer could have been expressed in terms of ℓ instead of r.

Consider a roller coaster with a circular loop of radius 100 m. A man is riding in the car sitting on a spring scale. When he and the car are completely upside down at the top of the loop, travelling at 50 m/s, what does the spring scale read if the man's mass is 50 kg? [Answer: 750 N, compared to a normal "weight" of 500 N.] Do you see how this is similar to the bucket of tennis balls, or bucket of water, swung in a vertical circle?