Kinematics of rigid body rotation:

\[ \omega(t) = \omega(0) + \alpha t. \]

\[ \theta(t) = \theta(0) + \omega(0) t + \frac{1}{2} \alpha t^2. \]

\[ \omega^2 = \omega(0)^2 + 2\alpha[\theta - \theta(0)]. \]

These expressions assume that the angular acceleration \( \alpha \) is a constant.

Note that \( \alpha \) and \( \omega \) are vectors, with both magnitude and direction.

The magnitudes are given (for rigid body rotation) by \( \omega = \Delta \theta / \Delta t \), and by \( \alpha = \Delta \omega / \Delta t \), as \( \Delta t \to 0 \)

Relation between Angular and Linear quantities:

\[ v_t = r\omega, \text{ and } a_t = r\alpha. \]

\[ a_r = \frac{v_t^2}{r} = r\omega^2. \]
How to get the acceleration $a$ of any point on a spinning rigid body:

$$a = a_r + a_t.$$

$$a_r = r\omega^2 \text{ and } a_t = r\alpha.$$

Therefore $a = r\sqrt{\omega^4 + \alpha^2}$.

If we define $\phi$ as the angle between $a$ and $a_r$, then we instantly see that

$$\phi = \tan^{-1}\left[\frac{a_t}{a_r}\right].$$
A rigid disc spinning about its center has $\alpha = -2 \text{ rad/sec}^2$ and $\omega(0) = 20 \text{ rad/sec}$. How far does it turn in the next 10 seconds? [100 rad] What is its angular speed after 10 seconds? [Zero.] What is its angular speed after 15 seconds? [−10 rad/sec]

A spinning rigid body has $\omega$ of 10 rad/sec, $\alpha$ of 80 rad/sec$^2$, at a certain instant, and we fix our attention on a point 1 m from the axis of spin. What is the acceleration $a$ of this point on the body, and what angle does it make with a line running from the point to the center of rotation? [The magnitude of $a$ is 128 m/s$^2$ and the angle it makes is 38.7° with a radius line.]

A car is rounding a curve with a radius of 200 m at a speed of 30 m/s. The mass of the car is 1000 kg. If the car is on the verge of skidding, what is the coefficient of static friction $\mu_s$ between tires and road? [0.45]
• Show that for a conical pendulum, with string of length \( l \) making an angle \( \theta \) with the vertical, thus moving in a circle of radius \( r = l \sin \theta \), the tension in the string is \( T = mg / \cos \theta \) and the constant speed of the pendulum is

\[
v = \sqrt{gr \tan \theta}.
\]

Note the answer could have been expressed in terms of \( l \) instead of \( r \).

Consider a roller coaster with a circular loop of radius 100 m. A man is riding in the car sitting on a spring scale. When he and the car are completely upside down at the top of the loop, travelling at 50 m/s, what does the spring scale read if the man’s mass is 50 kg? [Answer: 750 N, compared to a normal “weight” of 500 N.] Do you see how this is similar to the bucket of tennis balls, or bucket of water, swung in a vertical circle?