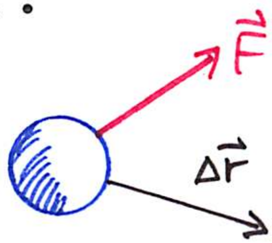
To work problems with Newton's 2nd Law, $\sum \mathbf{F} = m\mathbf{a}$, we need to know the forces. But if the forces vary in a complex way, changing direction and or magnitude with time, then the acceleration varies in an equally complex way, and the only procedure for discovering how the system will move is to set up a differential equation and solve it numerically step by step.

Isn't there some way to get a quick answer to how a system moves, without following it in detail, step by step? YES! Instead of using a dynamical equation, like the 2nd Law, we look for CONSERVATION LAWS.

WORK AND ENERGY!

Work done by a force:



$$W_F = F_{||}\Delta r,$$

Work is done only by the component of the force $\mathbf{F}$ parallel to the displacement $\Delta\mathbf{r}$.

Keeping track of work that shows up as acceleration:

$$\Delta W = \Delta K, \; where \; K = (1/2)mv^2.$$

Work done by non-conservative forces:

The work done by non-conservative forces, like friction, is path-dependent.

The work done by conservative forces, like gravity, is path-independent.

GRAVITATIONAL POTENTIAL ENERGY:

The work *we do* against gravity is accounted for by $\Delta W = PE_{gf} - PE_{gi} = mg(y_f - y_i)$, so we define $PE_g(y) = mgy$.

• A box is being slid across a horizontal tabletop by an applied force **F** making an angle θ with the horizontal. The box slides at constant velocity **v** a distance D across the tabletop, with kinetic friction acting. (a) How much work is done by the applied force **F**? (b) How much work is done by the net force acting on the box?

• A 1 kg ball travels along a complex, wiggly path in space. At one point it is moving tangent to the path at 5 m/s. Later it is moving in a totally different direction at 10 m/s. How much work was done on the ball by *all* the (unknown) forces acting on it, between these two points in space?

POTENTIAL ENERGY:

Potential energy can only be defined for a *conservative* force, because then the work done against the force does not depend on the path taken through space, but only on the beginning and ending points.

We define potential energy to increase *by the amount of work we have to do* to change the position of an object or the configuration of a system. Only the increase is defined, so the value of the potential energy at any point must be chosen arbitrarily. The choice in general has nothing to do with physics.

Gravitational potential energy near the surface of the earth can be obtained by considering the work done against gravity in lifting an object:

$$\Delta W = mg(y_f - y_i) = \Delta PE_g.$$

If gravity is the only force acting, $E = K + PE_g$ is a conserved quantity.

Spring potential energy can be obtained by estimating the work done to stretch a spring as $W_s = (1/2)k(x_f^2 - x_i^2)$. We can define the spring potential energy, therefore, as

$$PE_s = (1/2)k(x_f^2 - x_i^2).$$

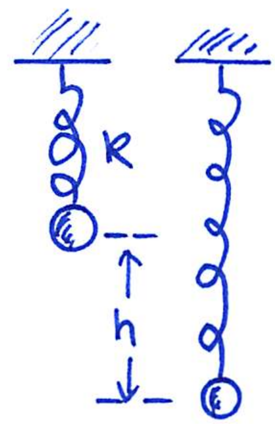
We often treat potential energy as a function, like $PE_g(y) = mgy$, or $PE_s(x) = (1/2)kx^2$. Note that these functions are arbitrary because the choice for where $x = 0$ and/or $y = 0$ is completely arbitrary.

Work done by non-conservative force:

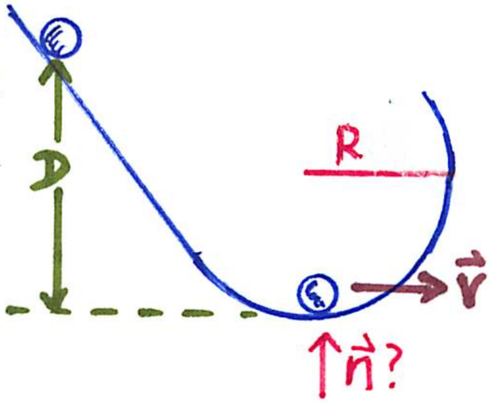
An object slides down a ramp onto a table, as kinetic friction acts on it. 2 m above the table it was moving at 10 m/s. When it slides onto the table it is moving at 9 m/s. If its mass is 1 kg, how much work did friction do on it?

Two potential energies!

An object of mass m is hung from an unstretched vertical spring of force constant k and allowed to fall from rest. When it comes to momentary rest again, how far has it fallen from its original position?



Using Conservation of Energy to supplement Newton's 2nd Law! An object rolls with negligible friction down a curved track as shown several times in class. If it starts from rest a distance D above the table the track is on, what supporting force does the track exert on it when it arrives at the very bottom of the track's circular arc of radius R?



CONSERVATION OF ENERGY:

If gravity is the only force doing work, $E = K + PE_g$ is conserved, it is the same number whatever point in space the object being studied happens to occupy.

WORK DONE BY NON-CONSERVATIVE FORCES:

$$W_{nc} = \Delta E.$$

SPRING POTENTIAL ENERGY:

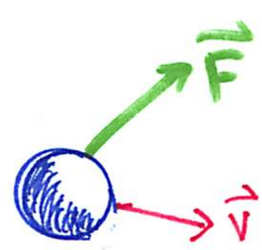
$$PE_s = (1/2)kx^2$$

is the work required to stretch a spring a distance x.

Generally:

$$E = \sum_i K_i + \sum_i PE_i.$$

POWER:



$$\mathcal{P}_{avg} = \frac{\Delta W}{\Delta t}.$$

Instantaneous power delivered by a force: $\mathcal{P} = F_{||}v$.

• A ball of mass 1 kg is pulled upward by a string, with a constant acceleration of 1 m/s^2, starting from rest. After the ball has been moving for 2 sec, what power is the string delivering to it? Answer: $\mathcal{P} = m(a+g)at$ which gives 22 Watts.

• A ball of mass 1 kg is being swung in a vertical circle at the end of a string of unknown length. It has a very different speed at the top and bottom of the circle, due to work done by gravity, but we are not told the speeds. If the tension in the string at the bottom of the circle is 70 N, is there any way to find the tension at the top?

Answer: Yes, if you use Conservation of Energy PLUS Newton's 2nd Law it is easy to show $T_b = T_t + 6mg$, so that the tension in the string at the top is 10 N.

Three Great Conservation Laws:

• ***Conservation of Energy*** is based on the fact that fundamental laws of physics are invariant under the transformation $t \rightarrow -t$.

• ***Conservation of Momentum*** is based on the fact that fundamental laws of physics are invariant under space translation, that is, under the change $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{b}$, where $\mathbf{b}$ is an arbitrary constant vector. (Any one point in space is as good as any other.)

• ***Conservation of Angular Momentum*** is based on the fact that fundamental laws of physics are invariant under space rotation. (Any one orientation in space is as good as any other.)

Physicists realized in the 20th Century that ALL fundamental laws could not be invariant under $t \rightarrow -t$, or otherwise the universe would not exist. You have to be able to get something from nothing, under certain conditions. Two huge accelerators called B-factories currently exist in order to study the couple of known processes that are described by laws not invariant under $t \rightarrow -t$. One big mystery is why MORE such processes are not seen. Failure of "time reversal invariance" should be fairly common, and instead it is extremely rare.

Quantum physics also indicates that Conservation of Energy cannot be required for processes that take place over an extremely short time, due to the famous Uncertainty Relation $\Delta E \Delta t \simeq \hbar$.