Center of Mass, Conservation of Momentum:

$$\mathbf{r}_{\rm cm} = \frac{\Sigma_i m_i \mathbf{r}_i}{\Sigma_i m_i}.$$

In a continuous approximation,

$$\mathbf{r}_{\rm cm} = \frac{1}{M}\int \mathbf{r}\, dM.$$

The importance of the center of mass is that *it is the point that obeys Newton's 2nd Law:*

$$\Sigma_i \mathbf{F}_i^{\rm ext} = M\mathbf{a}_{\rm cm}.$$

Note that if no external forces act on a system, $\mathbf{v}_{\rm cm}$ is a constant.

Total Momentum:

$$\mathbf{P} = \Sigma_i m_i \mathbf{v}_i = M\mathbf{v}_{\rm cm}.$$

CONSERVATION OF MOMENTUM:

Since

$$\Sigma_i \mathbf{F}_i^{\rm ext} = \frac{d\mathbf{P}}{dt},$$

This means that if no external forces act on a system, its total momentum $\mathbf{P}$ cannot change!

System Kinetic Energy:

The kinetic energy is a scalar so for a system of particles of mass m_i, we have

$$K = \frac{1}{2} \sum_i m_i v_i^2 .$$

What is more, we can partition the kinetic energies for a complex system, into a center of mass kinetic energy of the system as a whole, and the internal kinetic energy of the parts of the system relative to one another.

$$K = K_{\text{cm}} + K_{\text{int}} .$$

Impulse:

We can define

$$\Delta \mathbf{p} = \mathbf{J} = \int \mathbf{F}(t) dt.$$

For a force that acts for just a split second we can replace the function of t by an average over the short time interval Δt and write

$$\Delta \mathbf{p} = \mathbf{F}_{\text{avg}} \Delta t.$$

This is a very useful equation for estimating "impulsive" forces, that act only for a split second to change the momentum of an object. [For example, hitting a ball with a bat or racquet.]

Two Body Collisions:

(1) ELASTIC: The two bodies do no net work on one another, so $K_i = K_f$.

(2) INELASTIC: The two bodies do some work on one another, so $K_f < K_i$.

(3) COMPLETELY INELASTIC: The two bodies merge together or stick together during the collision.

For all three types of collisions, the total momentum vector *does not change* if no external forces do work.