Center of Mass, Conservation of Momentum:

\[ \mathbf{r}_{cm} = \frac{\Sigma m_i \mathbf{r}_i}{\Sigma m_i}. \]

In a continuous approximation,

\[ \mathbf{r}_{cm} = \frac{1}{M} \int \mathbf{r} dM. \]

The importance of the center of mass is that it is the point that obeys Newton's 2nd Law:

\[ \Sigma_i \mathbf{F}_i^{ext} = M \mathbf{a}_{cm}. \]

Note that if no external forces act on a system, \( \mathbf{v}_{cm} \) is a constant.

**Total Momentum:**

\[ \mathbf{P} = \Sigma_i m_i \mathbf{v}_i = M \mathbf{v}_{cm}. \]

**CONSERVATION OF MOMENTUM:**

Since

\[ \Sigma_i \mathbf{F}_i^{ext} = \frac{d\mathbf{P}}{dt}, \]
This means that if no external forces act on a system, its total momentum $\mathbf{P}$ cannot change!

**System Kinetic Energy:**

The kinetic energy is a scalar so for a system of particles of mass $m_i$, we have

$$K = \frac{1}{2} \sum_i m_i v_i^2.$$  

What is more, we can partition the kinetic energies for a complex system, into a center of mass kinetic energy of the system as a whole, and the internal kinetic energy of the parts of the system relative to one another.

$$K = K_{\text{cm}} + K_{\text{int}}.$$  

**Impulse:**

We can define

$$\Delta \mathbf{p} = \mathbf{J} = \int F(t)dt.$$  

For a force that acts for just a split second we can replace the function of $t$ by an average over the short time interval $\Delta t$ and write

$$\Delta \mathbf{p} = F_{\text{avg}} \Delta t.$$
This is a very useful equation for estimating "impulsive" forces, that act only for a split second to change the momentum of an object. [For example, hitting a ball with a bat or racquet.]

**Two Body Collisions:**

(1) **ELASTIC:** The two bodies do no net work on one another, so \( K_i = K_f \).

(2) **INELASTIC:** The two bodies do some work on one another, so \( K_f < K_i \).

(3) **COMPLETELY INELASTIC:** The two bodies merge together or stick together during the collision.

For all three types of collisions, the total momentum vector does not change if no external forces do work.