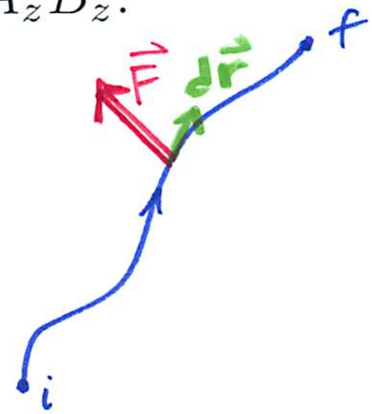
THE CONCEPT OF WORK!

- THE DOT PRODUCT:

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta_{AB}, \text{ or}$$

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z.$$



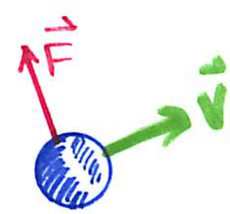
- Work done by a force $\mathbf{F}$:

$$W_F = \int_{\mathbf{r}_i}^{\mathbf{r}_f} \mathbf{F} \cdot d\mathbf{r}.$$

- Work done by a constant force $\mathbf{F}$:

$$W_F = \mathbf{F} \cdot \Delta\mathbf{r}.$$

- Power $\mathcal{P}$:



$$\mathcal{P} = \mathbf{F} \cdot \mathbf{v} \text{ and } \mathcal{P} = \frac{dW}{dt}.$$

$$W_F = \int \mathbf{F} \cdot \mathbf{v} dt.$$

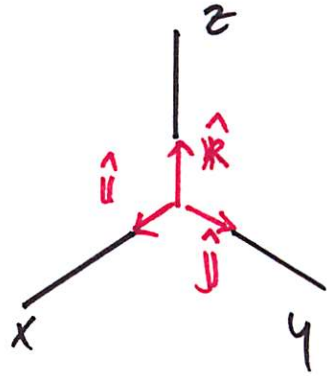
- Kinetic Energy:

$$K = \frac{1}{2}mv^2.$$

$$W_F = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2.$$

More on Dot Products:

Notice what the dot product among unit vectors leads to!



$$\widehat{\mathbf{i}} \cdot \widehat{\mathbf{j}} = \widehat{\mathbf{j}} \cdot \widehat{\mathbf{k}} = \widehat{\mathbf{k}} \cdot \widehat{\mathbf{i}} = 0.$$

But

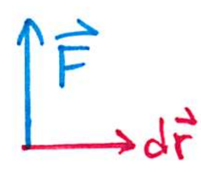
$$\widehat{\mathbf{i}} \cdot \widehat{\mathbf{i}} = \widehat{\mathbf{j}} \cdot \widehat{\mathbf{j}} = \widehat{\mathbf{k}} \cdot \widehat{\mathbf{k}} = 1.$$

Therefore if we have $\mathbf{A} = \widehat{\mathbf{i}}(2\text{ m}) + \widehat{\mathbf{k}}(4\text{ m})$, and $\mathbf{B} = \widehat{\mathbf{j}}(3\text{ m}) + \widehat{\mathbf{k}}(8\text{ m})$, then we can instantly see "by inspection" that

$$\mathbf{A} \cdot \mathbf{B} = 32\text{ m}^2.$$

When we calculate the work done by a particular force $\mathbf{F}$, namely

$$W_F = \int \mathbf{F} \cdot d\mathbf{r},$$



suppose we knew that $\mathbf{F} = \widehat{\mathbf{j}}[10\text{ Newtons}]$ and $d\mathbf{r} = \widehat{\mathbf{i}}dx$. Then it would be instantly obvious that $\mathbf{F}$ does no work, $W_F = 0$.

Some simple examples of work done:

- Varying force— Suppose $\mathbf{F} = \widehat{\mathbf{i}}bx^2 + \widehat{\mathbf{j}}cx$, where b and c are constants. How much work is done by this force if an object moves from $x = 0$ to x_f along the x axis?

Answer: $W_F = \int \mathbf{F} \cdot d\mathbf{r}$. In this case $d\mathbf{r} = \widehat{\mathbf{i}}dx$. So taking the dot product gives

$$W_f = \int_0^{x_f} bx^2 dx = b{x_f}^3/3.$$

- Spring:

For a spring, say, $\mathbf{F}_s = -\widehat{\mathbf{i}}kx$. If we pull along x with a force which balances this, and moves the right end of the spring a distance x from equilibrium, with the left end fixed, the same argument as we went through before gives

$$W_s = (1/2)kx^2.$$

Note this is the work *we do to stretch the spring,* not the work the spring does. The work the spring does is $-W_s$.

- Gravity:

Suppose we exert a force $\mathbf{n}$ to lift an object of mass m, on the surface of the earth, a vertical distance

y_f starting from $y = 0$. Since we are supporting the object as we lift it, if the net force on the ball is $\mathbf{n} + m\mathbf{g} = 0$, then $n = mg$ in magnitude, and since the force is constant the work done from $W_n = \mathbf{n} \cdot \Delta\mathbf{r}$ is obviously

$$W_g = mgy_f.$$

This is *the work we do against gravity, in lifting.*