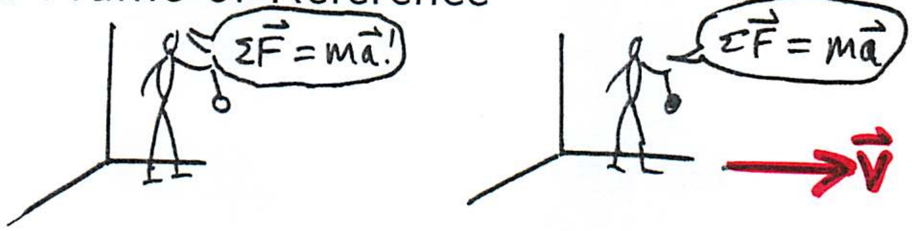
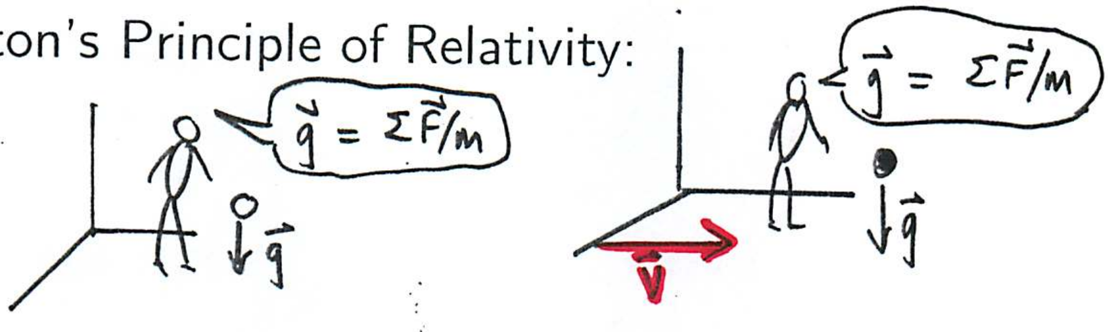
RELATIVITY!

- Inertial Frame of Reference



Find a coordinate system in which you can verify that all the laws of physics work. That's called an "inertial frame of reference." Any coordinate system moving at *constant velocity* **v** with respect to the first system *is also an inertial frame of reference.*

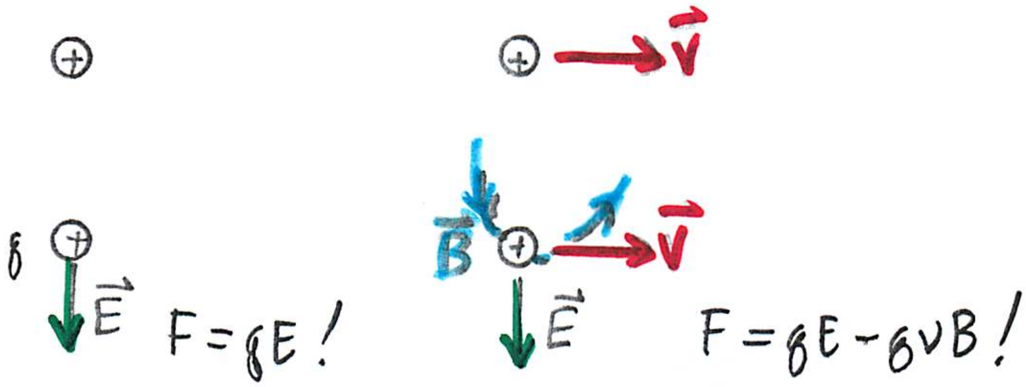
- Newton's Principle of Relativity:



The laws of physics are the same in any and all inertial frames, and any calculation using the laws *must give the same result* in any inertial frame.

- Maxwell's Equations Violate Newtonian Relativity!

A moving charge does not exert the same force on another moving charge that the charge would exert if both were at rest!



The force in one case has magnitude qE_x, for example, and in the other frame it has magnitude $q[E_x - v_y B_z]$!

So who is wrong, Newton or Maxwell? Experiments conducted in the late 19th Century tended to suggest the strong possibility that both sets of laws are wrong under certain conditions. Einstein, in the first few years of the 20th Century, searched for a new approach that would allow one to write laws that are accurate under any and all circumstances.

EINSTEIN'S POSTULATES:

- The speed of light is a universal constant, the same in all inertial frames of reference. It is a property of empty space itself and has nothing to do with motion of source or observer.

- The laws of physics have the same form in all inertial frames of reference. Physicists usually call this form "covariant," so the challenge is how to write all laws of physics in a covariant form.

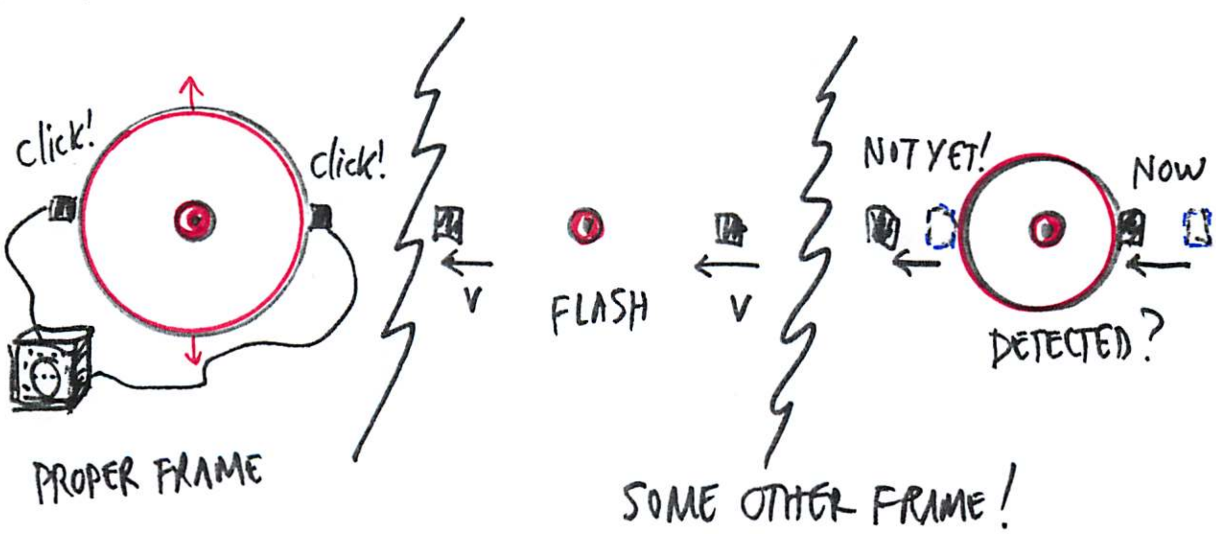
We can use the postulates to explore various consequences of "thought experiments," which in turn suggest new laws of physics, or old laws expressed in a totally different mathematical framework.

• RELATIVITY OF SIMULTANEITY

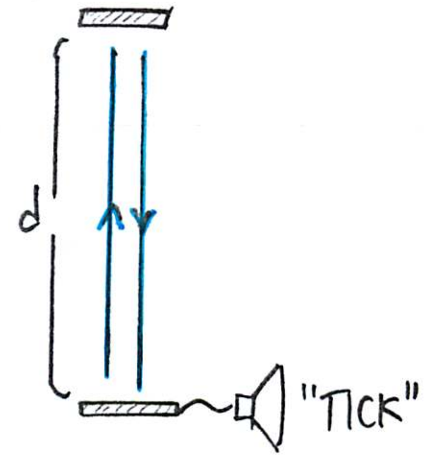
If the speed of light (which is also the speed with which information propagates from one point in a system to another) is independent of the motion of source or observer, then we instantly see that two events or processes which occur simultaneously in one inertial frame, but happen at different points in space, cannot be simultaneous in any other inertial frame.

If the processes are moving with respect to the observer, the leading or approaching process always occurs before the trailing or receding process.

Thus no law of physics can depend upon observers agreeing that events or processes occur at the same time.



Pulse Clock:
Proper time:



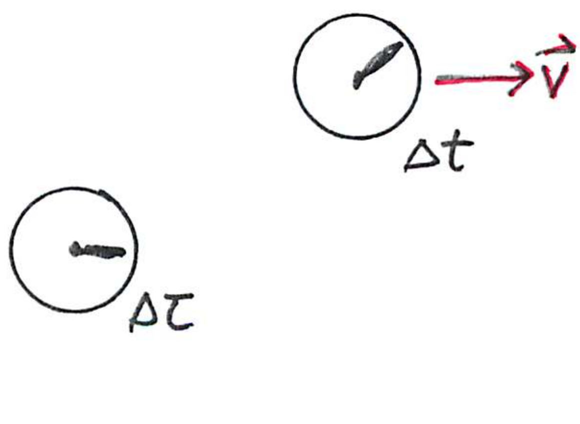
$$\Delta\tau = \frac{2d}{c}.$$

From any other frame:

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-(v/c)^2}}.$$

Note that *for any non-zero v*, $\Delta t > \Delta\tau$. **All moving processes run slow compared to the identical processes at rest.** The proper time interval is always the shortest interval associated with the process.

Thus a process which takes 1 sec in the proper frame might take 2 sec in another inertial frame. This frame would be travelling at $\sqrt{1-(v/c)^2} = 1/2$ or $v = 0.866c$.



Disagreement on Distances:

Suppose a spaceship travelling at v with respect to two stars a distance L apart takes time Δt to make the trip, from a viewpoint in which the two stars are at rest. Then clearly the time taken is $\Delta t = L/v$.

But to the crew of the ship, the voyage takes the proper time $\Delta\tau = \Delta t\sqrt{1-(v/c)^2}$. Thus we are forced to conclude that seen from the ship, the distance between the stars is

$$L' = L\sqrt{1-(v/c)^2}.$$

The stars are moving past the ship at speed v and the time elapsed between passages by the two stars is $\Delta\tau$, so seen from the ship the stars are closer together.

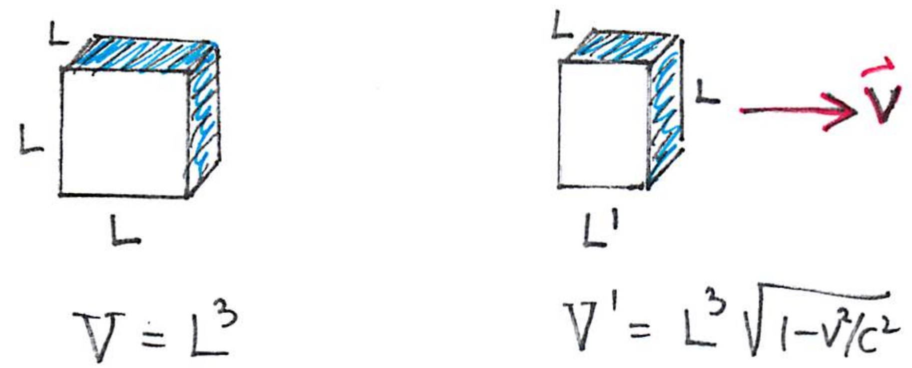
Disagreement on Volume!

This effect shows up only in distances measured along the line of motion. So consider a cube which in its rest frame has volume $V = L^3$. If the cube is moving parallel to one side, at speed v then its volume becomes $V' = L'L^2 = V\sqrt{1-(v/c)^2}$.

Notice that although the mass of the cube is not different, *its density is larger!*

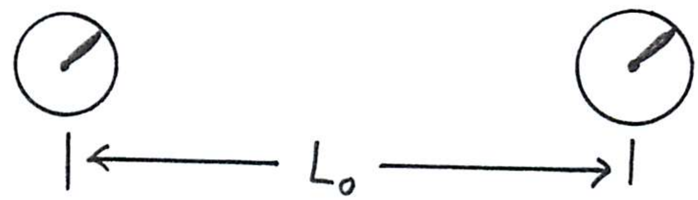
$$\rho = M/V \text{ so } \rho' = M/V' = \frac{\rho}{\sqrt{1-(v/c)^2}}.$$

Thus mass and charge densities are larger for objects moving with respect to us, than for the same objects at rest, although the total mass and total charge are precisely the same.



Another important realization:

Processes taking place at two different locations, which are simultaneous in their mutual proper frame are not simultaneous viewed from any other frame.

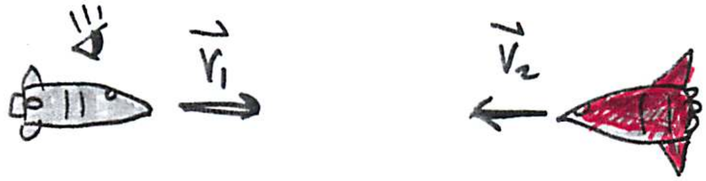


Therefore, if we have two synchronized clocks a distance L_0 apart in their proper frame, when we view the clocks from a frame in which they are moving at some speed v, we find the discrepancy between the readings of the two clocks is

$$\Delta t = \frac{L_0 v}{c^2}.$$

The approaching process is always ahead of the trailing process by this amount.

Velocity Addition:



In 317K you learned, for a situation like this, that the speed of 2 seen from 1 is

$$v_{21} = v_1 + v_2.$$

This result is only a rough approximation. The exact result is

$$v_{21} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}.$$

Example: $V_1 = V_2 = 0.9c$

$$\therefore V_{21} = \frac{1.8c}{1 + 0.81} = 0.99c\,!$$

•RELATIVISTIC MOMENTUM:

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-(v/c)^2}}.$$

•RELATIVISTIC KINETIC ENERGY:

$$KE = mc^2\left[\frac{1}{\sqrt{1-(v/c)^2}} - 1\right].$$

Note that

$$\frac{v}{c} = \sqrt{1-(mc^2/E)^2},$$

where

$$E = KE + mc^2.$$

Note also that

$$E^2 = p^2c^2 + m^2c^4.$$

Also:

$$\frac{v}{c} = \frac{pc}{E}.$$

Four-Vectors: In order to have the same form in all inertial frames of reference, laws of physics must be written in terms of "four-vectors." For example, the energy-momentum 4-vectors are

$$p^\mu = [E/c, -\mathbf{p}] \text{ and } p_\mu = [E/c, \mathbf{p}].$$

The inner product is invariant:

$$p_\mu p^\mu = (E/c)^2 - p^2 = (mc)^2.$$

For example we could define $x^\mu = [ct, -\mathbf{r}]$ and then $x^\mu x_\mu = (ct)^2 - r^2 = 0$, for example, in all inertial frames of reference. [Imagine a pulse of light emitted by a source. In any inertial frame the radius of the wave front is given by $r = ct,\ r' = ct'$, etc.]

Thus relativistic versions of all classical Newtonian laws, like the 2nd Law of Motion, Conservation of Momentum, etc., have to be expressed in terms of 4-vectors in order to be correct in general.

•BINDING ENERGY:

Einstein pointed out that the mass of a system at rest is

$$Mc^2 = \sum_i K_i + \sum_i V_i + \sum_i m_i c^2,$$

where the system is composed of individual pieces m_i held together with potential energies V_i and having kinetic energies K_i. But for essentially all systems in nature the size of an atomic nucleus or larger, the potential energies are negative (attractive forces) and the overall internal energy $\mathcal{E} = K_{\text{tot}} + V_{\text{tot}}$ is therefore negative.

This means that $M < \sum_i m_i$, that is, in general, the mass of a system *is less than* the mass of the pieces the system is made of!

Physicists define the binding energy $B = -\mathcal{E}$. This is the work it would take to completely disassemble the system.

- A star ship is sent from our solar system to the Bunn system, 20 c-years distant. Ignoring time to speed up and slow down, if the star ship cruises at $v/c = 0.866$, how long does the trip take according to the passengers, and what distance do they travel in that time?
- Coker in the late 1970s and early 1980s did experiments with protons ($m_p c^2 = 940$ MeV) that had a kinetic energy of 700 MeV, coming out of a linear accelerator called LAMPF. What was v/c for these protons?

A particle of mass M initially at rest converts into two photons. If the mass of the particle is 130 MeV/c^2, what is the momentum of each of the photons?

Answer: 65 MeV/c.

Four particles, each of mass 940 MeV/c^2, form a bound system. The mass of the system is measured to be 3720 MeV/c^2. What is the binding energy of the system?

Answer: 40 MeV.

EINSTEIN'S THEORY OF GRAVITATION:

It is based on two postulates:

(1) All the laws of physics must have the same form in all frames of reference, either inertial or non-inertial.. (Thus the approach is sometimes called "General Relativity.")

(2) The effects of acceleration are completely indistinguishable in a local environment from the effects of gravity.

Therefore Einstein concluded that anything acceleration can do, gravity can also do.

Consequences of the approach:

(1) Gravity can be replaced by space-time curvature with the understanding that objects always take the shortest path (geodesic) in curved space-time.

(2) Photons are affected by gravity. Anything with kinetic energy is affected by gravity, even if the thing is massless. Thus, the *gravitational red shift,* and the *gravitational effect on time intervals.*

(3) The *gravitational lens*, indispensable to modern astronomers.

(4) The Global Positioning System!