\[ \vec{B} = \frac{k \mu \vec{g}'}{r^2} (\vec{v}' \times \vec{r}) \]

\[ \vec{F} = g (\vec{v} \times \vec{B}) \quad \text{or} \quad \vec{F} = g (E + \vec{v} \times \vec{B}) \]

\[ d\vec{F} = I \, dl \times \vec{B} \]
Force on Currents:

Start with $d\mathbf{F}_m = dq \mathbf{v} \times \mathbf{B}$. Since a current is along the path of a wire, we can write $\mathbf{v} = d\mathbf{l}/dt$, so that

$$d\mathbf{F}_m = (dq/dt)d\mathbf{l} \times \mathbf{B}.$$  

This gives us a general differential expression for the force on a current in a magnetic field:

For a long, straight wire, obviously

$$\mathbf{F} = I L \times \mathbf{B}.$$  

In general we have to integrate over the wire shape to find the net force:

$$\mathbf{F} = I \int d\mathbf{l} \times \mathbf{B}.$$
If we have a loop of current in a $\mathbf{B}$ field it's easy to see that there may be an unbalanced torque.

The torque has to be calculated from the definition,

$$\vec{\tau} = \vec{r} \times \mathbf{F},$$

looking at each segment of the loop and summing to get the net torque.

The torque generally depends in a natural way on the magnetic moment,

$$\vec{\mu} = I \int d\mathbf{A}.$$  \hspace{1cm} (1)

In fact,

$$\vec{\tau} = \vec{\mu} \times \mathbf{B}.$$  \hspace{1cm} (2)

The magnetic moment is as fundamental a quantity as charge and mass. Fundamental particles have a specific, intrinsic magnetic moment just as they have specific, intrinsic charge, mass and angular momentum!
Susceptibility \[ \chi = \frac{B_{\text{mat}}}{B_{\text{vac}}} \].

**Ferromagnetism:** \( \chi \) is of the order of thousands, as magnetic moments of electrons align with the field and enhance it. Nickel, iron and cobalt are the standard ferromagnetic materials.

**Paramagnetism:** \( \chi \) is very tiny. The majority of materials are paramagnetic.

**Diamagnetism:** \( \chi \) is small and *negative*. Superconductors, carbon, silver, bismuth, mercury and lead are famous diamagnets.

\[ \mu = \mu_0 [1 + \chi]. \]

**HALL EFFECT:**

\[ \Delta V_H = \frac{IB}{enL}. \]
• A long wire is pulled up into a square form with one open side, and the other three sides of length \( \ell \). If the ends of the wire have length \( L \) and the wire is in a magnetic field out of the plane of the partial square, what is the net force on the wire? [Magnitude and direction.]

The forces on the sides of the partial square cancel, so only the part parallel to the rest of the wire experiences a net force. So the total force is \( \mathbf{F} = 2I(\mathbf{L} \times \mathbf{B}) + I(\mathbf{\ell} \times \mathbf{B}) \). The direction is in the plane of the page toward the bottom, and has magnitude \( F = I(2L + \ell)B \).

• Put a metal core in a solenoid, with the core having \( \chi = 3000 \). The solenoid carries a 2 A current and has \( n = 10^5 \) per meter. What is \( B \) along the central axis of the solenoid?

If the solenoid had no core, \( B = \mu_0 n I \). When a core is present, we replace \( \mu_0 \) by \( \mu = \mu_0 (1 + \chi) \). Thus \( B = \mu_0 (1 + \chi) n I \), and plugging in the numbers gives 754 T.

• If the Hall effect is used to find the number of charge carriers per unit volume in a certain metal, it is found for a current of 1 Amp, in a magnetic
field of 5 T, for a sample of width \( d \) and length \( L \), so that the area of the plate is \( A = dL \), the Hall voltage works out to be \( \Delta V_H = (IB)/(neL) \). What therefore is \( n \) when \( L \) is \( 10^{-4} \) m and \( \Delta V_H = 10^{-5} \) V?

We see \( n = IB/(eL\Delta V_H) \). Here of course \( e \) is the magnitude of the electron charge. Plugging in the numbers results in \( n = 3 \times 10^{28} \) per cubic meter.

- A magnetic dipole \( \vec{\mu} \) is in a nonuniform magnetic field that varies along the \( x \) axis. If \( \vec{\mu} \) is also along \( +x \) what is the resulting force exerted on the dipole?

\[
W_{\text{ext}} = F_{\text{ext}} \Delta x = \Delta U = -\mu [B(x + \Delta x) - B(x)].
\]

Here we used \( U = -\vec{\mu} \cdot \vec{B} \). So the net force on the dipole is approximately

\[
F_{\text{ext}} = -\mu [B(x + \Delta x) - B(x)]/\Delta x = -\mu (dB/dx).
\]

This is the force we would have to exert to move the dipole around \textit{against} the force exerted by the field, so since \( \mu \) is not a function of position,

\[
F_B = + (d/dx)(\vec{\mu} \cdot \vec{B}).
\]

In the general case, we wind up with the result that should have been obvious from the first:

\[
\mathbf{F}_B = \nabla (\vec{\mu} \cdot \vec{B}) = -\nabla U.
\]
In class we wrote the force in a slightly different way, again taking advantage of the fact that $\bar{\mu}$ is not a function of position.