

Study Guide for Ch. 29:

Maxwell's Equations describe electromagnetic radiation as oscillating electric and magnetic fields, in a vacuum, propagating at a fixed speed given by $c = 1/\sqrt{\mu_0\epsilon_0}$, which is about 3×10^8 m/s. Quantities like c , and h (Planck's Constant) are universal constants of nature which set the fundamental scales of all physical phenomena.

If in an electromagnetic wave the peak value of the oscillating electric field is E_p , then the corresponding peak value of the oscillating magnetic field is $B_p = E_p/c$. The frequency of the source of the EM waves, f , determines the wavelength of the waves: $\lambda = c/f$.

Maxwell showed that an electromagnetic pulse of total energy U carries a momentum of magnitude $p = U/c$.

The direction in which the wave transports energy and momentum is given by the Poynting Vector,

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}.$$

The unit of \mathbf{S} is power per unit area. If we take the time average of \mathbf{S} over one cycle of oscillation, the result is usually just called the wave intensity, I .

There are many different ways to express I in terms of either E_p or B_p or E_{rms} or B_{rms} , using the relation of μ_0 to ε_0 and the speed of light, and the relation between E_p and B_p . Just as one example,

$$I = \langle S \rangle = \frac{E_p^2}{2\mu_0 c} = \frac{cB_p^2}{2\mu_0}.$$

Remember that the energy per unit volume in an electromagnetic wave, based on our discussion of energy storage in capacitors and inductors, is

$$u = (1/2) \left[\varepsilon_0 E^2 + \frac{B^2}{\mu_0} \right].$$

An EM wave can exert pressure on a surface it impacts. To see what the pressure is, consider $F/A = (1/A)dp/dt$ where the vectors are projected along the direction of \mathbf{S} . Since $p = U/c$, we get $F/A = [1/(Ac)]dU/dt = [1/(Ac)]\mathcal{P}$, where \mathcal{P} is the power transported. But in time average this is IA . Thus we get a pressure $F/A = I/c$. And I/c is just the energy density, u . As you probably learned in 317K, the correct unit for pressure is energy per unit volume, so this makes perfect sense. Note also that if the wave were perfectly reflected, the pressure (using

conservation of momentum) would be twice as great as if the wave were completely absorbed.

This chapter briefly discusses the idea of polarization. Since the intensity of the wave goes like E^2 , if we take an initially unpolarized wave and put it through a system which projects out only a single direction for \mathbf{E} , we have a polarized wave. If we then take this polarized wave and send it through another system which projects out another single direction, making an angle of θ with the original direction of polarization, then the intensity of the wave as it leaves the second system is reduced by $\cos^2 \theta$.

In the early 20th Century it was shown both experimentally and theoretically that light actually consists of particles, called photons. Photons are the first known example of a class of particles called *bosons*, which are distinguished by not obeying conservation of particle number. Photons are chargeless and massless, but carry kinetic energy and momentum. The kinetic energy of each photon created by a charge oscillating at frequency f is $K = hf$, where h is Planck's constant, and the momentum of each of those photons is given by $p = K/c$. h in convenient units is 4.1×10^{-15} eV-sec, and the standard corresponding unit of momentum is an eV/c.