Maxwell's Equations:

$$\oint \mathbf{E} \cdot d\mathbf{A} = q/\epsilon_0.$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0.$$

$$\oint \mathbf{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}.$$

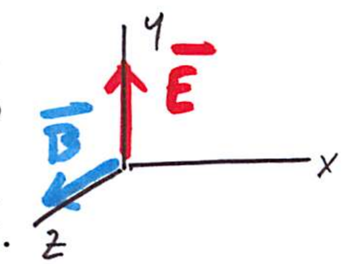
$$\oint \mathbf{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{A}.$$

These equations have solutions even when q and I are zero.

Accelerated charge emits electromagnetic radiation fields, which propagate outward at the speed of light. For continuous harmonic acceleration at angular frequency ω, for a wave propagating down $+x$, we get

$$\mathbf{E}(x,t) = E_p \sin[kx - \omega t]\widehat{\mathbf{j}},$$

$$\mathbf{B}(x,t) = B_p \sin[kx - \omega t]\widehat{\mathbf{k}}.$$



These expressions are solutions of Maxwell's equations (see pp. 513 - 515) if $\omega/k = 1/\sqrt{\epsilon_0 \mu_0} = c$, the observed speed of light, and $E_p = cB_p$.

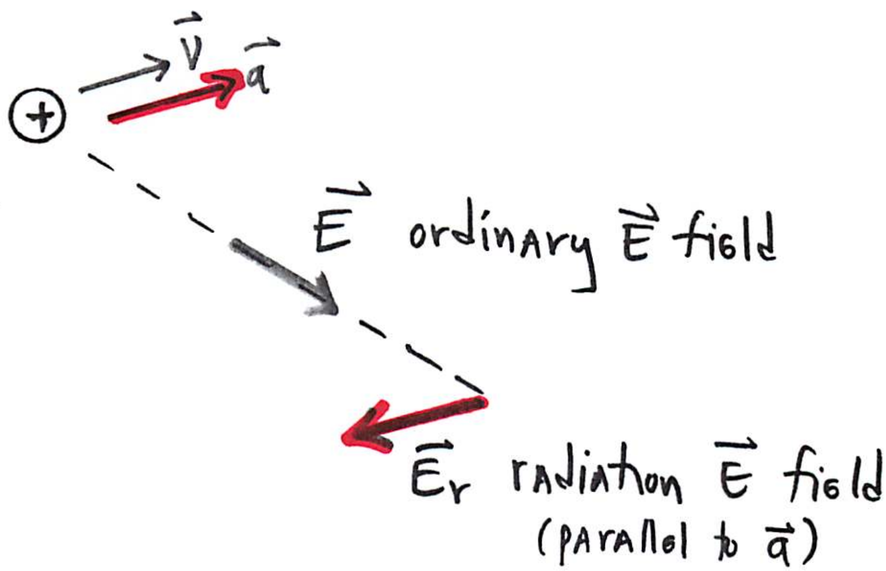
The triumph of 19th Century physics...

James Clerk Maxwell wrote down four famous equations that summarized everything learned about electric and magnetic fields by the early 1860s. He found that these equations had a solution that predicted a totally new combination of electromagnetic fields, produced when charges *accelerate*. These fields are wavelike and move with a speed $c = (\sqrt{\epsilon_0 \mu_0})^{-1}$, the speed of light! That is $c = \lambda f$.

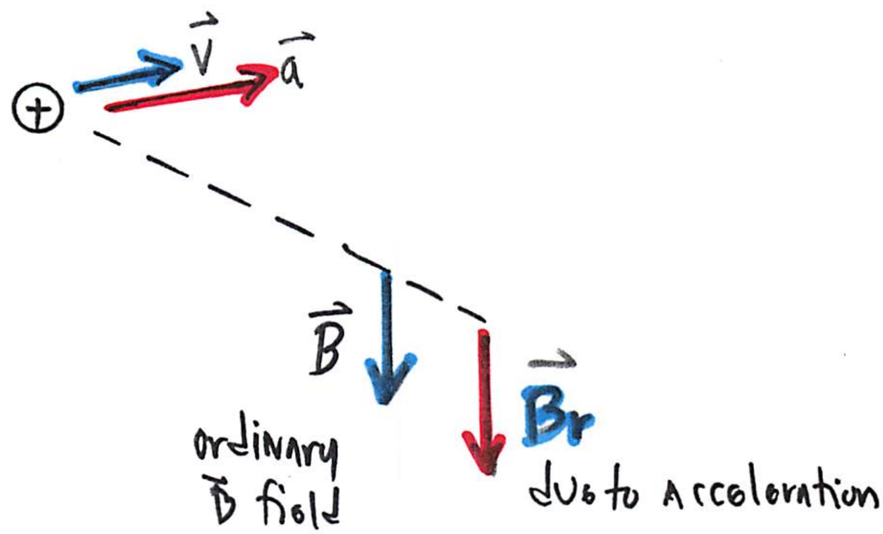
But what were these fields? A transverse propagating disturbance in things made up by Faraday is not a very satisfying physical picture!

In 1905, Einstein showed that the electromagnetic field has to be made of particles, which he called *photons.* By 1930 physicists knew that all fundamental fields are made of particles, called *bosons.* But if the field is made of particles what is "waving'?

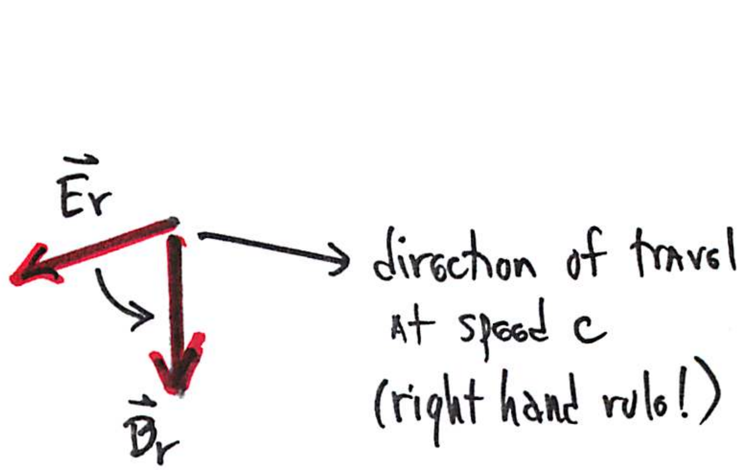
In the 1920s Max Born showed that in the new quantum physics, any particle with definite energy and momentum is described by *a wave of probability.* Thus the astonishing result is that electromagnetic waves are "made of probability." We will discuss this more toward the end of the course. It means that where the wave is most intense, you will find most of the photons.



$\vec{v}$
$\vec{a}$
$\vec{E}$ ordinary $\vec{E}$ field
$\vec{E}_r$ radiation $\vec{E}$ field
(parallel to $\vec{a}$)



$\vec{v}$
$\vec{a}$
$\vec{B}$
$\vec{B}_r$
ordinary
$\vec{B}$ field
due to acceleration



$\vec{E}_r$
direction of travel
at speed c
(right hand rule!)
$\vec{B}_r$

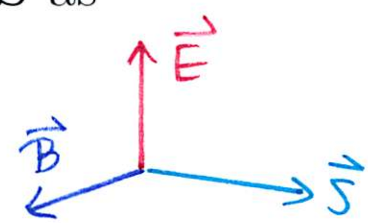
An EM wave is said to be polarized if $\mathbf{E}$ lies along a specific axis.

EM waves transport both kinetic energy and momentum!

$$S = \mathcal{P}/A = (c/2)[\epsilon_0 E + B/(\mu_0)].$$

We can define the Poynting Vector $\mathbf{S}$ as

$$\mathbf{S} = \frac{1}{\mu_0}[\mathbf{E} \times \mathbf{B}].$$



It has units of Watts per meter^2.

We need to average S over a wave cycle of oscillation. The result can be written many ways:

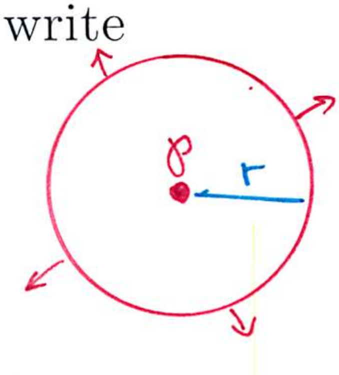
$$< S >= \frac{E_p B_p}{2\mu_0} = \frac{E_{\rm rms} B_{\rm rms}}{\mu_0}.$$

or

$$< S >= \frac{E_p^2}{2\mu_0 c} = \frac{c B_p^2}{2\mu_0}.$$

For a point source, the waves are outgoing with spherical maxima and minima, so we can write

$$< S >= \frac{< \mathcal{P} >}{4\pi r^2}.$$



Maxwell proved that EM waves carry momentum, given by:

$$p = U/c.$$

Thus, if EM waves are absorbed by a surface, they exert a pressure on the surface. If they are reflected by the surface, the pressure is doubled.

PHOTONS:

Electromagnetic radiation consists of particles called (by Einstein) *photons.* Photons have no charge and no mass, but can carry kinetic energy and momentum. If we have a beam of visible light which has frequency f and wavelength λ, the photons have kinetic energy

$$K = hf,$$

where the fundamental constant $h = 6.6 \times 10^{-34}$ J-s or 4.14×10^{-15} eV-s. They have momentum

$$p = h/\lambda = K/c,$$

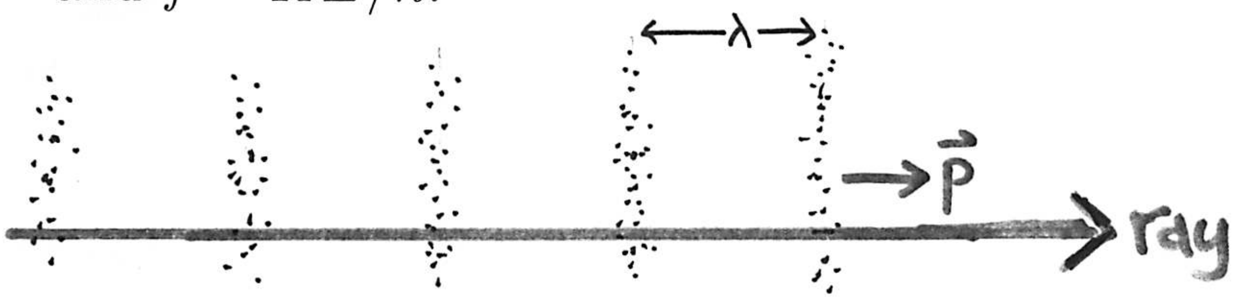
where c is the speed of light in a vacuum, about 3×10^8 meters per sec.

The usual unit of momentum used by physicists is the eV/c. A photon with a K of 2.4 eV thus has a momentum of 2.4 eV/c.

RAY OPTICS AND REFRACTION

All forms of light consist of photons, particles which are created when a charge is accelerated. If the charge is accelerated with frequency f the photons have $KE = hf$, where h is Planck's Constant, and momentum $p = KE/c$. The typical KE of a photon of visible light is several eV.

Light appeared to be waves when studied in the 18th through 19th Centuries because quantum particles like the photon, electron, etc., have probability distributions in space which are wavelike, with $\lambda = h/p$ and $f = KE/h$.



WAVES VERSUS RAYS:

In drawing sketches, it is far easier to draw ***rays***, which are lines perpendicular everywhere to the wave fronts, instead of a qualitative wave. After all, the actual waves of visible light have wavelengths of less than a millionth of a meter, which would not be possible to draw for processes on a human scale.