Because the average of the square of a sinusoidal function of the form $\cos \omega t$ or $\sin \omega t$ is 1/2, it is conventional to define "root mean square" values to replace an oscillator amplitude. Thus if we have

$$V(t) = V_p \sin[\omega t + \phi] \text{ and } I(t) = I_p \sin[\omega t + \phi],$$

we define

$$V_{\text{rms}} = \frac{V_p}{\sqrt{2}} \text{ and } I_{\text{rms}} = \frac{I_p}{\sqrt{2}}.$$

Resistance Only:

$I = V/R = (V_p/R)\sin(\omega t) = I_p \sin(\omega t)$, so $I_{\text{rms}} = V_{\text{rms}}/R$.

Capacitance Only:

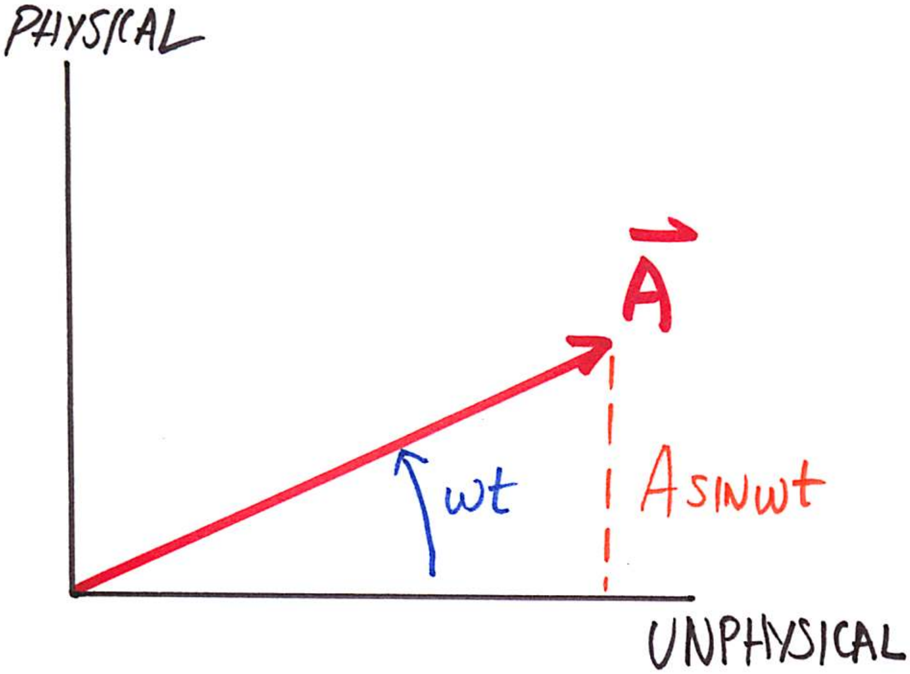
$$I = C\omega V_p \cos(\omega t) = I_p \sin(\omega t + \pi/2).$$

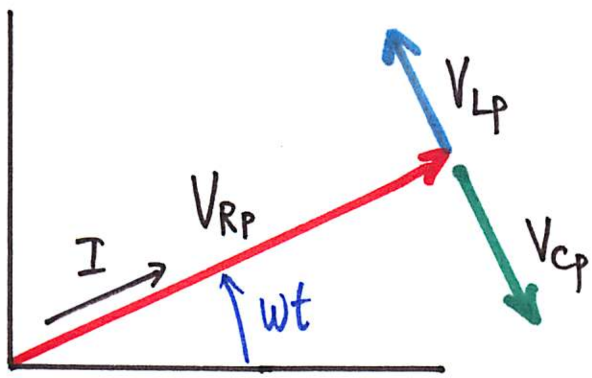
Note that if $X_c = 1/(\omega C)$ then $I_p = V_p/X_c$, and similarly for the rms values.

Inductance Only:

$I = -(V_p/\omega L)\cos(\omega t)$ so $I = (V_p/X_L)\sin[\omega t - \pi/2]$.

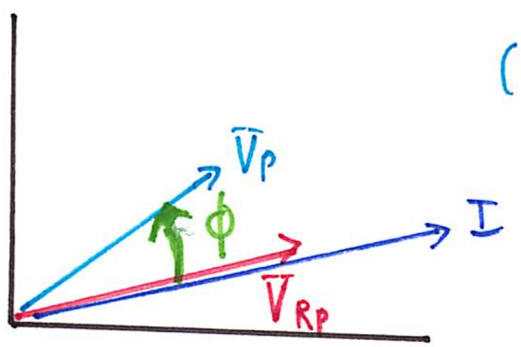
This means $I_p = V_p/X_L$ with $X_L = \omega L$.





Define Z by $I_p Z = V_p$

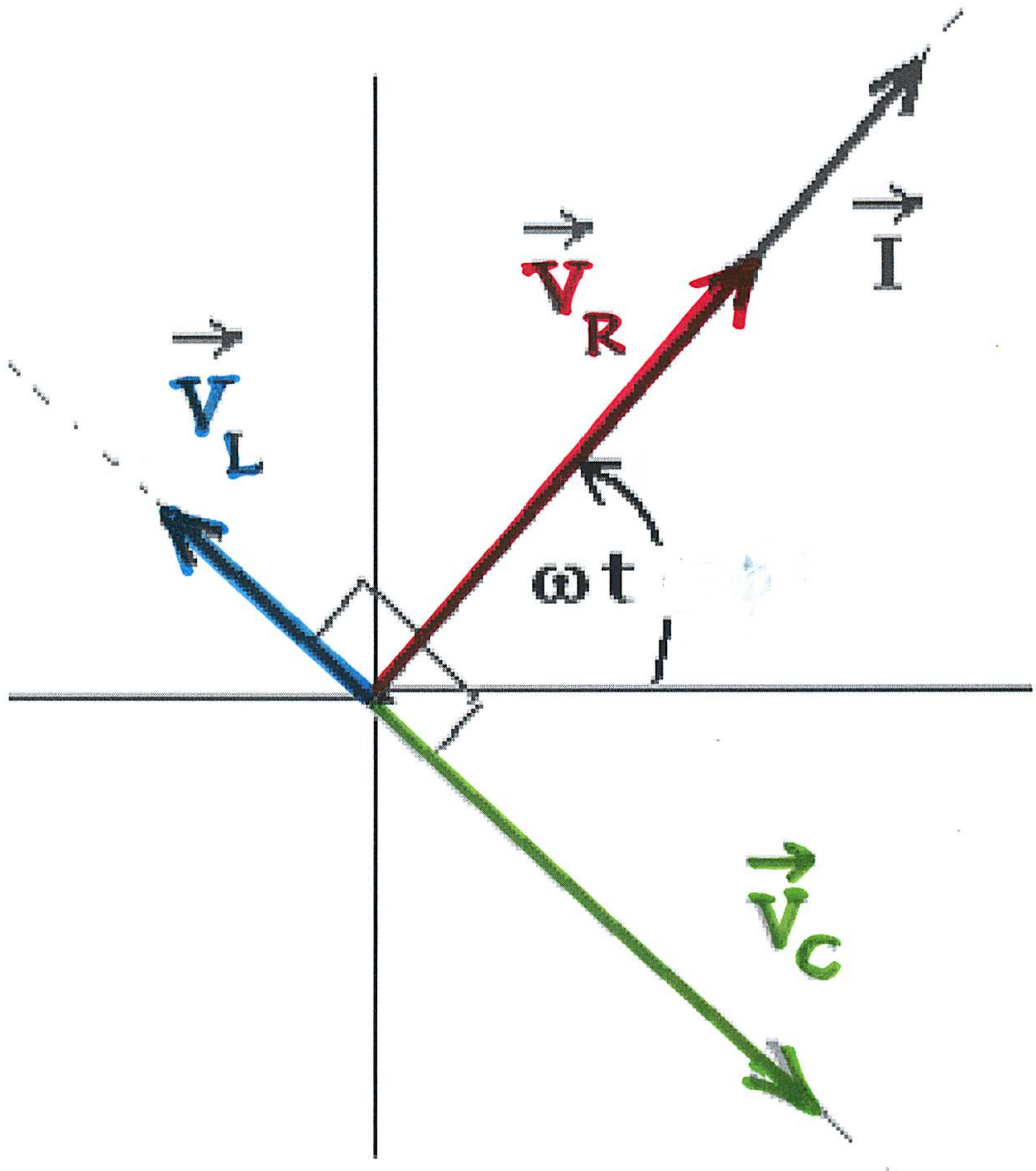
Then $Z = \sqrt{R^2 + (X_L - X_C)^2}$



$(\vec{V}_p = \vec{V}_{Rp} + \vec{V}_{Lp} + \vec{V}_{Cp})$

ϕ, phase angle between I and $\vec{V}_p$

$\tan\phi = (X_L - X_C)/R$!



RLC Phasor

SEE page 493 in text!

Phasors are a graphical way to get the same insights that use of complex numbers and functions (having both real and imaginary parts) supplied in the early 20th Century.

LC Oscillator A pure LC circuit is described by $L(d^2q/dt^2)+q/C = 0$, so $q(t) = q_p \cos \omega t$, $\omega = 1/\sqrt{LC}$.

LRC Oscillator:

$$q(t) = q_p \exp(-Rt/2L) \cos \omega t.$$

Critical damping occurs when $2L/R = 1/\omega = \sqrt{LC}$.

Resonance: From $V_p = \sqrt{V_{Rp}^2 + (V_{Lp} - V_{Cp})^2}$ it is easy to see that

$$I_p Z = V_p, \text{ where } Z = \sqrt{R^2 + (X_L - X_c)^2}.$$

Clearly the maximum current is seen when $X_L = X_c$, or $\omega = 1/\sqrt{LC}$.

Phase angle between current and emf

$$\tan \phi = \frac{X_L - X_c}{R}.$$

Delivered average power:

$$< \mathcal{P} >= I_{\rm rms} V_{\rm rms} \cos \phi.$$

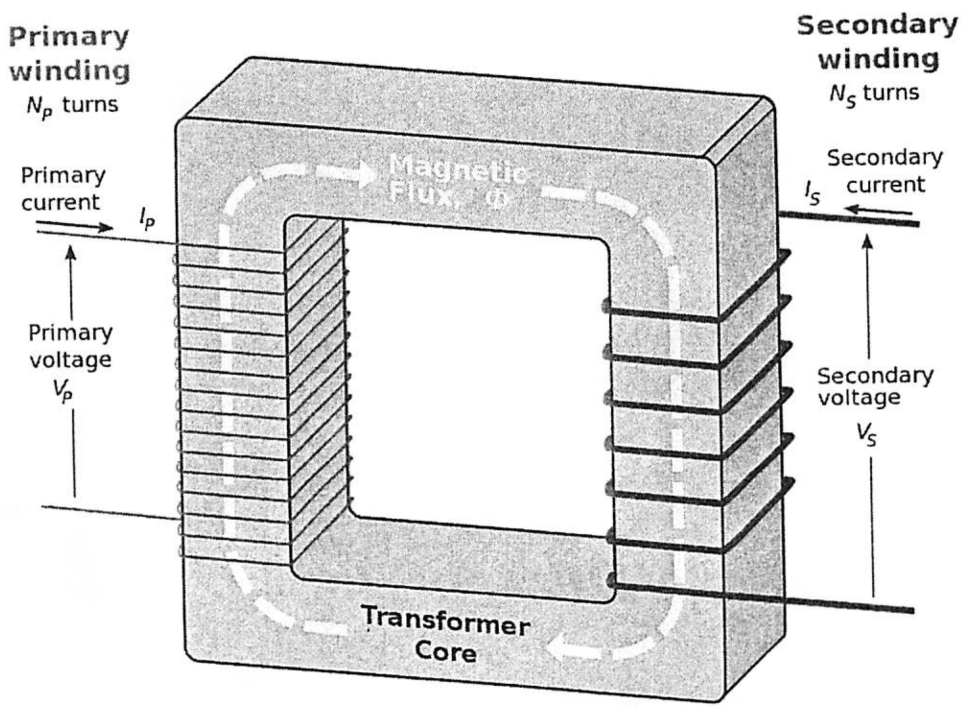
Transformer:

$$I_1 V_1 = I_2 V_2 \text{ but } \frac{V_2}{V_1} = \frac{N_2}{N_1}.$$

Since the power input equals the power output, ignoring resistance losses, then

$$I_1 \Delta V_1 = I_2 \Delta V_2.$$

By varying the windings of the two sides of a transformer one can use it to "step down" or "step up" voltage. Supplying of electricity by utilities would not be possible without transformers... usually there is one on every block of every street in every city!



In an LRC AC circuit we have an angular frequency of 1 rad/sec, a resistance of 1 Ohm, an inductance of 2 H, a capacitance of 1 F and V_p is 10 V. Find f, Z, I_p, ϕ, V_{pR}, V_{pL} and V_{pC}.

Answers: $f = 0.16$ Hz, $X_L = 2\ \Omega$, $X_c = 1\ \Omega$ and $Z = \sqrt{2}\ \Omega$. Thus I_p is 7.1 Amps, ϕ is 0.785 rad or 45 degrees, and the voltages are, respectively 7.1 V, 14.2 V, and 7.1 V. Why do they NOT sum up to 10 V, by the way??

Suppose an LRC circuit has a frequency of $50/\pi$ Hz with an emf amplitude of 40 V, an inductance of 160 mH, a capacitance of 99 μF, and a resistance of 68 Ohms. What is I_p, what is ϕ, what is $I(t)$, and what is the earliest time for which I is zero? What average power is being delivered by the emf? What is the resonant angular frequency? What is I_p at resonance?

We first need to calculate ω, X_L, X_C, and then Z. We find $Z = 109$ Ohms. Then I_p is 0.37 Amps, and ϕ is -0.896 rad. Thus $I(t) = [0.37A]\sin[(100\ \mathrm{s}^{-1})t + 0.896]$. Thus the earliest time at which the current is zero is 0.022 sec. Using the equation for $\mathcal{P}_{\mathrm{avg}}$ we get 4.6 Watts. The resonant frequency is 251 rad/sec or 40 Hz. At that frequency, I_p is 0.59 A.