Definition of magnetic flux:

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}.$$

Faraday's Law of Induction:

$$\mathcal{E} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}.$$

Note that there are a huge number of totally different ways to change the flux: you could change the strength of **B** with time, or the direction of **B** with time, or the magnitude or orientation of **A** with time!

The Lenz Law: The direction of the induced emf, induced current, and resulting **B** field are always in such a direction as to OPPOSE the change in flux that created them. [Conservation of Energy; someone or something has to do work to create the emf.]

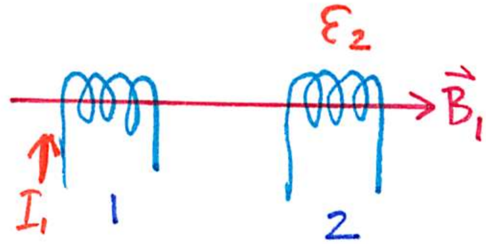
Electric generators were invented by Faraday. A generator is basically an electric motor run "backward"!

Eddy currents are induced in conductors by flux changes.

Note: flux changes through a closed conducting path will induce a current around the path. But flux changes across a conductor that is not part of a circuit will induce a *potential difference* across the conductor.

Mutual Inductance:

$$\mathcal{E}_2 = -M_{21}\frac{dI_1}{dt}.$$



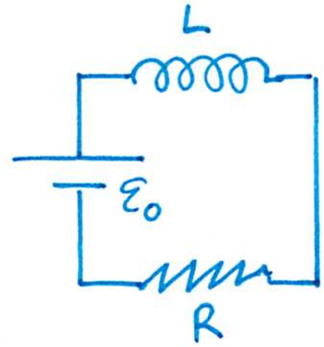
Self Inductance:

$$L = \Phi_B/I \text{ so } \mathcal{E} = -L\frac{dI}{dt}.$$

The induced emf always opposes any flux change, and is often called a "back emf."

Circuit with L and R: When an emf $\mathcal{E}_0$ is placed across a series LR circuit,

$$\mathcal{E}_L = -\mathcal{E}_0 \exp(-Rt/L)$$



and

$$I = \frac{\mathcal{E}_0}{R}[1 - \exp(-Rt/L)].$$

Energy stored in the magnetic field of an inductor:

$$U = (1/2)LI^2.$$

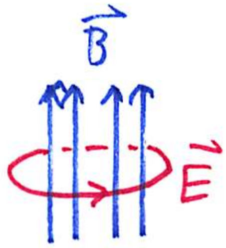
Energy stored in any magnetic field:

$$u_B = \frac{B^2}{2\mu_0}.$$

Compare to $u_E = (1/2)\epsilon_0 E^2$.

Induced Electric Field:

$$\oint \mathbf{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{A}.$$



This ought to remind you of Ampere's Law, and can be used in systems with cylindrical symmetry very easily. Notice that this induced **E** field is *nonconservative* and cannot be described in terms of potential energy, because a conservative field would have to satisfy

$$\oint \mathbf{E} \cdot d\vec{\ell} = 0.$$

Working problems where a changing magnetic field induces an electric field in empty space:

The basic law is

$$\oint \mathbf{E} \cdot d\vec{\ell} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}.$$

The one subtlety to keep in mind in the law is that the directions of $d\mathbf{A}$ and $d\vec{\ell}$ satisfy a right-hand rule. That is, if you choose $d\mathbf{A}$ to be in a certain direction, then $d\vec{\ell}$ has to wrap around that direction like the fingers of your right hand wrap around your thumb.

We only consider examples with cylindrical symmetry, where only $\mathbf{B}$ is varying, in otherwise empty space. We take $d\vec{\ell}$ to be parallel to the direction we assume for $\mathbf{E}$. What is that direction? It is the direction in which *current would be induced in a conducting loop lying along that path.*

Then by symmetry, at some point r measured from the center of the magnetic field region, $\oint \mathbf{E} \cdot d\vec{\ell}$ would just equal $2\pi r E$. And the integral over $\mathbf{B}$ would be over the area *inside that loop.* This ought to remind you of Ampere's Law, because it looks the same and works the same!