Current:

$$I = \frac{dQ}{dt}.$$

The current is taken to "flow" in the direction of $\mathbf{E}$ within a conductor across a potential difference. The unit of I is the Amp, a Coulomb per second.

Microscopic classical view: $I = qnAv_d$, where the charge is q, the number of charges per unit volume is n, the area of the conductor is A, and the "drift speed" of the charges through the conductor is v_d. For typical currents, v_d is around 10^{-4} m/s.

Current density: $\mathbf{j}$ is the current per unit area. $\mathbf{j} = nq\mathbf{v}_d$.

Conductivity: σ is defined by $\mathbf{j} = \sigma\mathbf{E}$.

Resistivity: $\rho = 1/\sigma$ so $\mathbf{j} = \mathbf{E}/\rho$. If we define a Volt per Amp as an Ohm (Ω), then the unit of ρ is Ω-m.

The resistivity depends strongly on temperature, and over the vast range of solids, namely conductors, semiconductors and insulators, it has an incredible range from about 10^{-8} up to 10^{17} Ω-m!

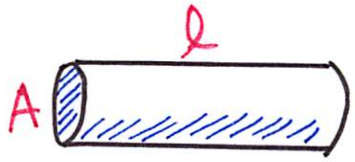
Ohm's Rule: For conductors, to a fair approximation, $I = V/R$, where R is defined as the resistance in Ohms.

In general, $I = \int \mathbf{j} \cdot d\mathbf{A}$.

For a uniform $\mathbf{j}$, we have $I = \mathbf{j} \cdot \mathbf{A} = VA/(\rho\ell) = V/(\rho\ell/A)$. Here we used $V = E\ell$.

This means we can calculate the resistance of a conductor of length ℓ and area A by

$$R = \frac{\rho\ell}{A}.$$



Power: $\mathcal{P} = IV = I^2R = V^2/R$. This power is dissipated as heat in the resistor, due to energy lost when electrons collide with atoms in the solid.

The charge flowing in a conductor for $t > 0$ is $q(t) = at^3 + bt + c$, where the constants have values of 4 C/s^3, 5 C/s and 6 C respectively. What is the current at $t = 1$ sec? What is the current density at that time, if the wire has an area of 2 cm^2?

In a copper wire, with $\rho = 1.7 \times 10^{-8}$ Ω-m, the electron density per unit volume is 8.48 $\times 10^{28}$ per cubic meter. If the electron drift speed is 6 mm/s, what is the electric field in the copper wire?

A tungsten wire has a resistance of 19 Ohms at 20°C and a resistance of 140 Ohms at a much higher T. If the function $R(T)$ is linear over this range and α is 0.0045/K, what is the temperature T?

A 160 km long wire from a power plant carries a current of 1000 A at 200 kV. If the resistance of the wire is 0.31 Ω/km, how much power is wasted by heating the wire?

A resistor dissipates 0.5 W at 3 V. How much will it dissipate at 1 V?

Some additional examples:

• A wire has resistivity of 7×10^{-8} Ω-m, is 10^2 m long and has area 0.1 mm^2. If there is a 10 V potential difference along its length, what is the current through it? Answer: $I = VA/(\rho\ell) = 0.14$ Amps.

• A current of 5 Amps flows through a 100 Ohm resistor for 1 hour. How much total heat was generated over that hour? Answer: 9×10^6 Joules!

• The current density in a wire of radius r_0 is given by $j(r) = a(r_0 - r)$, where a is a constant. What is the value of a in terms of the total current in the wire? Answer: by integration of $I = \int \mathbf{j} \cdot d\mathbf{A}$ we find $a = (3I)/(\pi r_0^3)$.