Current:

\[ I = \frac{dQ}{dt}. \]

The current is taken to “flow” in the direction of \( \mathbf{E} \) within a conductor across a potential difference. The unit of \( I \) is the Amp, a Coulomb per second.

**Microscopic classical view:** \( I = qnAv_d \), where the charge is \( q \), the number of charges per unit volume is \( n \), the area of the conductor is \( A \), and the “drift speed” of the charges through the conductor is \( v_d \). For typical currents, \( v_d \) is around \( 10^{-4} \) m/s.

**Current density:** \( \mathbf{j} \) is the current per unit area. \( \mathbf{j} = nqv_d \).

**Conductivity:** \( \sigma \) is defined by \( \mathbf{j} = \sigma \mathbf{E} \).

**Resistivity:** \( \rho = 1/\sigma \) so \( \mathbf{j} = \mathbf{E}/\rho \). If we define a Volt per Amp as an Ohm (\( \Omega \)), then the unit of \( \rho \) is \( \Omega \)-m.

The resistivity depends strongly on temperature, and over the vast range of solids, namely conductors, semiconductors and insulators, it has an incredible range from about \( 10^{-8} \) up to \( 10^{17} \) \( \Omega \)-m!

**Ohm’s Rule:** For conductors, to a fair approximation, \( I = V/R \), where \( R \) is defined as the resistance in Ohms.

In general, \( I = \int \mathbf{j} \cdot d\mathbf{A} \).
For a uniform $\mathbf{j}$, we have $I = \mathbf{j} \cdot \mathbf{A} = VA/(\rho \ell) = V/(\rho \ell / A)$. Here we used $V = E \ell$.

This means we can calculate the resistance of a conductor of length $\ell$ and area $A$ by

$$R = \frac{\rho \ell}{A}.$$

**Power:** $P = IV = I^2 R = V^2 / R$. This power is dissipated as heat in the resistor, due to energy lost when electrons collide with atoms in the solid.
The charge flowing in a conductor for \( t > 0 \) is \( q(t) = at^3 + bt + c \), where the constants have values of 4 C/s\(^3\), 5 C/s and 6 C respectively. What is the current at \( t = 1 \) sec? What is the current density at that time, if the wire has an area of 2 cm\(^2\)?

In a copper wire, with \( \rho = 1.7 \times 10^{-8} \ \Omega\cdot\text{m} \), the electron density per unit volume is \( 8.48 \times 10^{28} \) per cubic meter. If the electron drift speed is 6 mm/s, what is the electric field in the copper wire?

A tungsten wire has a resistance of 19 Ohms at 20°C and a resistance of 140 Ohms at a much higher T. If the function \( R(T) \) is linear over this range and \( \alpha \) is 0.0045/K, what is the temperature T?

A 160 km long wire from a power plant carries a current of 1000 A at 200 kV. If the resistance of the wire is 0.31 \( \Omega/\text{km} \), how much power is wasted by heating the wire?

A resistor dissipates 0.5 W at 3 V. How much will it dissipate at 1 V?
Some additional examples:

- A wire has resistivity of $7 \times 10^{-8} \, \Omega\cdot\text{m}$, is $10^2 \, \text{m}$ long and has area $0.1 \, \text{mm}^2$. If there is a 10 V potential difference along its length, what is the current through it? Answer: $I = VA/(\rho\ell) = 0.14 \, \text{Amps}$.

- A current of 5 Amps flows through a 100 Ohm resistor for 1 hour. How much total heat was generated over that hour? Answer: $9 \times 10^6 \, \text{Joules}$!

- The current density in a wire of radius $r_0$ is given by $j(r) = a(r_0 - r)$, where $a$ is a constant. What is the value of $a$ in terms of the total current in the wire? Answer: by integration of $I = \int j \cdot dA$ we find $a = (3I)/(\pi r_0^3)$.