Energy stored in a system of charge:

$$U = \sum_{i,j;\ i<j} \frac{k q_i q_j}{r_{ij}}.$$

Example:

$$U = k\big[q_1 q_2/r_{12} + q_1 q_3/r_{13} + q_2 q_3/r_{23}\big].$$

Capacitance

$$C = \frac{Q}{V}. \text{ example } C = \frac{\epsilon_0 A}{d}.$$

Stored Energy = Work to Charge:

$$U = (1/2)QV = (1/2)CV^2 = (1/2)(Q^2/C).$$

Effect of Dielectrics:

$$C = (\kappa \epsilon_0 A)/d = \kappa C_0.$$

Parallel Capacitors:

$$C_{\text{tot}} = \sum_i C_i.$$

Series Capacitors:

$$C_{\text{tot}}^{-1} = \sum_i C_i^{-1}.$$

Energy stored in an electric field:

The energy per unit volume at a point where the field is **E** is

$$u_E = (1/2)\epsilon_0 E^2.$$

How much work would it take to place 100 μC of charge onto a spherical isolated conductor, to bring its surface to a potential of 100 Volts?

The work required would be the total potential energy at the end, which is $U = (1/2)QV$. Plugging in the numbers should give you 0.005 Joules.

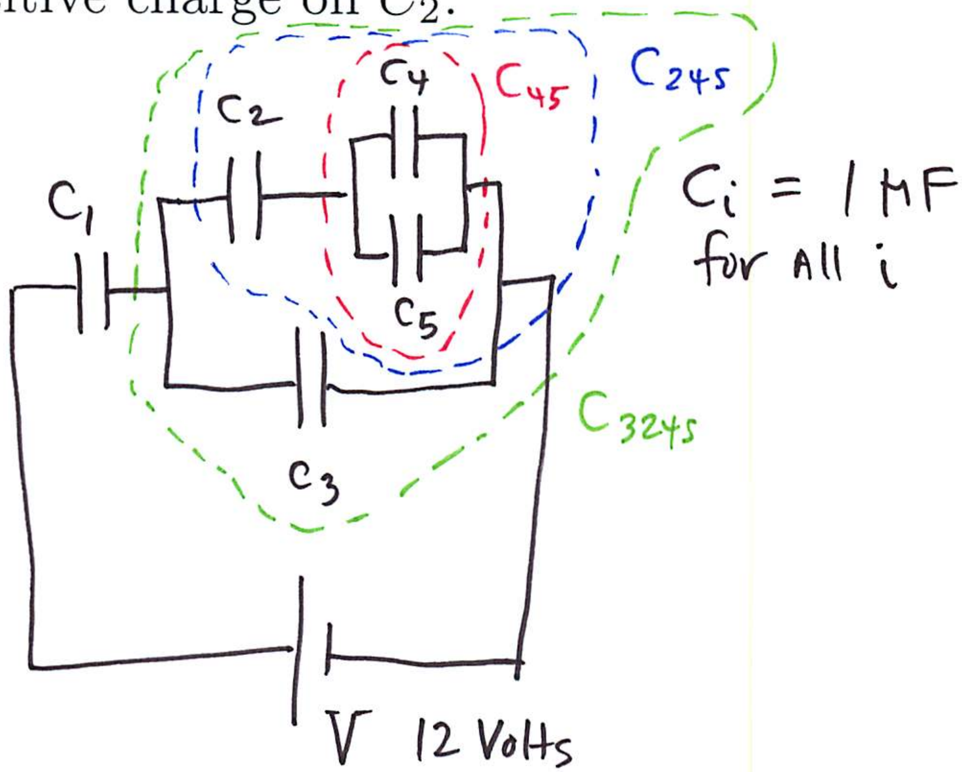
That is, 0.5 times 100 $\times 10^{-6} \times 100 = 0.5 \times 10^{-2}$ J.

Suppose capacitors C_5 and C_4 are in parallel and are placed in series with C_2 and then that arrangement is placed in parallel with C_3 and then the entire arrangement is put in series with C_1 and hooked across the terminals of a 12 V battery. If all the capacitors are identical, with capacitance of 1 μF, find

(a) The total capacitance of the system.

(b) The potential difference across C_3.

(c) The positive charge on C_2.



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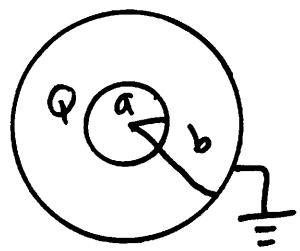
A 2 μF capacitor is charged by a 12 V battery and a 3 μF capacitor is charged by a 6 V battery. The capacitors are then removed from the batteries and connected in series so that the + plate of each capacitor is hooked directly to the − plate of the other. When the system comes to equilibrium, what is the + charge on each capacitor, what is the potential drop across each capacitor, and by how much did the total energy stored in the two capacitors change when they were hooked to one another?

A 10μF capacitor with a $\kappa = 10$ dielectric inside is charged by a battery and then disconnected when its positive plate has 10 μC of charge. How much work would be required to pull the dielectric out of this isolated capacitor? [Note that pulling the dielectric out cannot change the charge on the plates!] Answer: the work done is about 4.5×10^{-5} J.

Now, suppose the capacitor is kept connnected to the battery as the dielectric is pulled out. Is the work required the same or different, and why? [Hint: now the PD across the capacitor can't change!]

A hollow metal sphere of radius a is given a total charge Q. It is then surrounded by a larger hollow metal sphere of radius b, initially uncharged. The outer sphere is then grounded. What is the resulting potential difference between the two spheres? What is the total energy stored in the **E** field between the spheres? Assume $a = 2$ m and $b = 4$ m, with $Q = 4.0\ \mu$C.



Capacitance:

$$C = Q/V.$$

For a conducting sphere with charge Q, radius R, the capacitance is $C = R/k = 4\pi\varepsilon_0 R$.

Unit: Farad = Cou/Volt.

Typical capacitances, 1μF, 10^{-6} F,

1 nF, 10^{-9} F, and

1 pF, 10^{-12} F.

Parallel Plates

$$C = \frac{\epsilon_0 A}{d}.$$

In general, $C = Q/[-\int \mathbf{E} \cdot d\vec{\ell}]$.

Battery:

Fixed, chemically created potential difference.