VECTOR— a quantity with both a *magnitude* and a *direction*.

ONE-DIMENSIONAL MOTION:

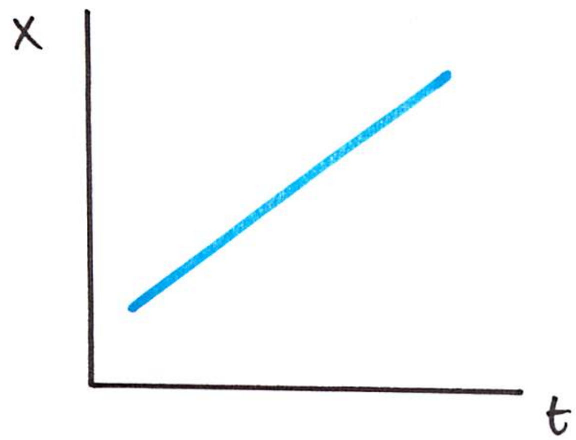
Average velocity: $\bar{v} = (v_f + v_i)/2$

But also:

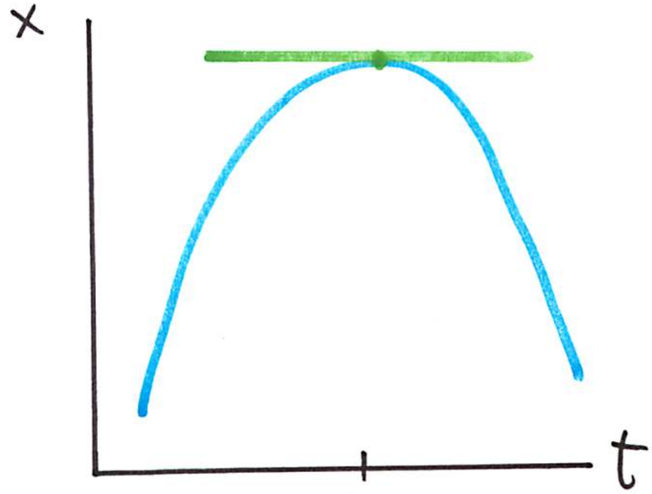
$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}.$$

Instantaneous velocity: $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$

What this means is that at each point $x(t)$, the instanteous velocity $v_x(t)$ is the slope of the function $x(t)$ at the time t! This is vitally important to understand!



Vx constant for all t!



$V_x(t_1) = 0!$

Average Acceleration:

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}.$$

Instantaneous Acceleration: $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$.

Important! If v_x is taken as positive, then if we also find a_x positive, it means the object is speeding up.

If v_x is taken as positive, then if we find a_x negative, it means the object is slowing down.

In other words, the instantaneous $a_x(t)$ *is the slope* of the curve $v_x(t)$ at any time t!

Being able to draw $v_x(t)$ qualitatively from a glance at $x(t)$ and being able to draw $a_x(t)$ qualitatively from a glance at $v_x(t)$ is a skill that students of 302K need to pick up as fast as possible.

MOTION AT CONSTANT ACCELERATION:

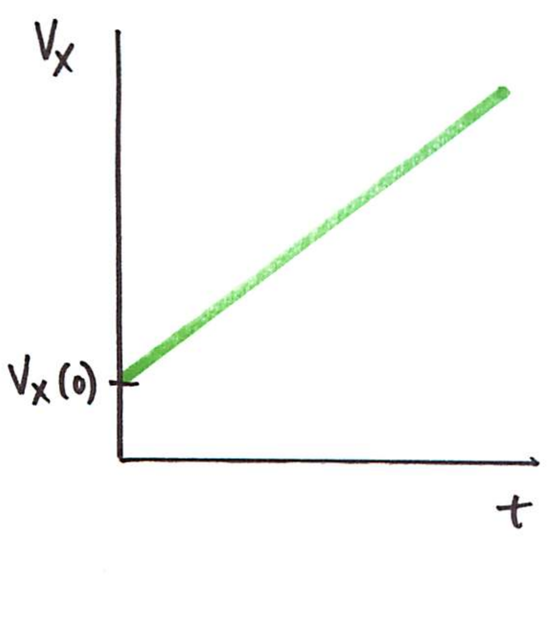
$$v = v_0 + at.$$

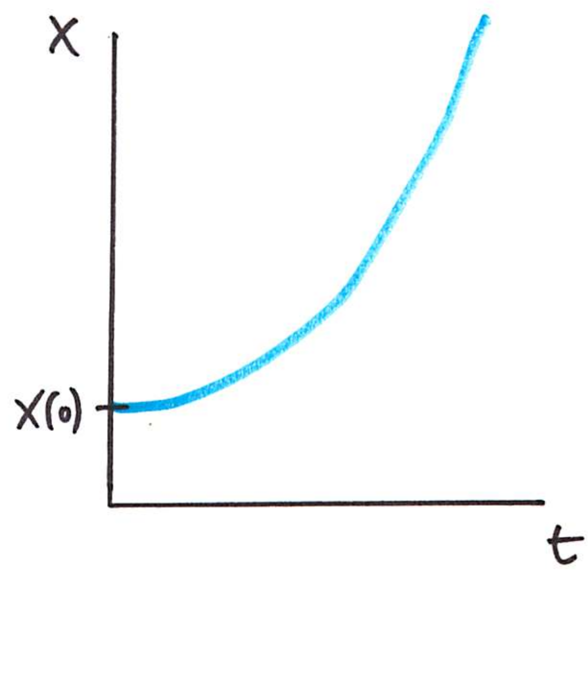
$$x = x_0 + v_0 t + (1/2)at^2.$$

$$v^2 = v_0^2 + 2a(x - x_0).$$

Note that the slope of $v_x(t)$ does not change because a_x does not change.

The slope of $x(t)$ is constantly increasing because v_x is constantly increasing.



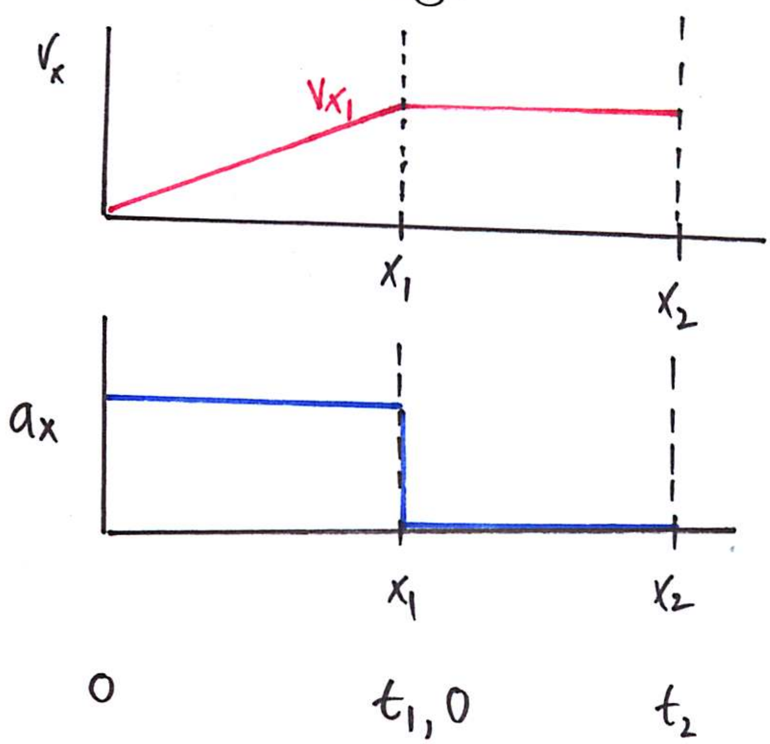


A toy car starts from rest and accelerates at 1 m/s^2 over a distance of 50 m. It then coasts at constant velocity for 15 sec, and then stops instantaneously when it strikes a wall.

(a) For what total time was the toy in motion?

(b) What was the total distance travelled to the wall?

(c) Where was the car at $t = 6$ sec, and how fast was it moving?



Free Fall:

For constant acceleration along y,

$$y(t) = y(0) + v_y(0)t + (1/2)a_y t^2,$$

$$v_y(t) = v_y(0) + a_y t, \text{ and}$$

$$v_y^2 - v_y(0)^2 = 2a_y(y - y(0)).$$

For free fall, $a_y = -g$. Thus,

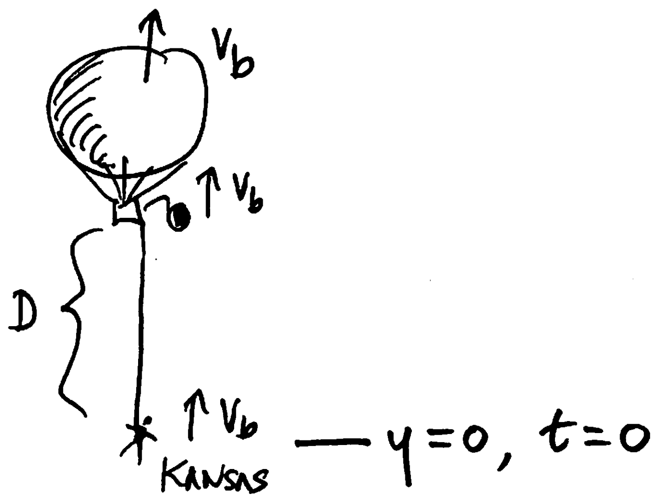
$$y(t) = y(0) + v_y(0)t - (1/2)gt^2.$$

$$v_y(t) = v_y(0) - gt, \text{ and}$$

$$v_y^2 - v_y(0)^2 = -2g(y - y(0)).$$

A ball is tossed upward at 30 m/s. How high does it go above the point of release? How long does it stay in the air if the tosser catches it? With what velocity does it return to the hand?

A balloon containing a bad guy is rising at a constant 10 m/s. Hanging 100 m below the balloon basket on a rope is famous adventurer Kansas City Smith. If the bad guy drops a sandbag, hoping to knock Kansas off, at what time does the sandbag reach his position?



A helicopter is hovering 100 m above the ground. If a bag of supplies is thrown straight down from it at an initial speed of 20 m/s, how long does it take to hit the ground?

Remember that the general solution(s) to

$$ax^2 + bx + c = 0$$

is (are)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Algebraic Solutions?

At $t = 0$ ball 1 is a distance D below ball 2 and their upward velocities are v_1 and v_2. If $v_1 > v_2$ the balls will collide at some point y_c. Assume ball 1 is at $y = 0$ at $t = 0$. Show that:

$$y_c = \frac{v_1 D}{v_1 - v_2}\left[1 - \frac{gD}{2v_1(v_1 - v_2)}\right].$$

Solution: We begin with $y_1 = v_1 t - (1/2)gt^2$ and $y_2 = D + v_2 t - (1/2)gt^2$. Setting $y_1 = y_2$ and solving for t_c, the time at which this happens, we get the simple result $t_c = D/(v_1 - v_2)$.

We can now plug this time into either one of the equations, for y_1 or y_2. Doing the algebra and cleaning up will give

$$y_c = \frac{v_1 D}{v_1 - v_2}\left[1 - \frac{gD}{2v_1(v_1 - v_2)}\right].$$

Notice that there is no solution if $v_1 = v_2$... the balls never collide at any finite y_c.