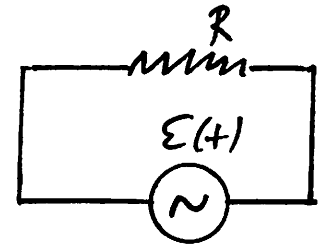
Alternating Current Circuits

RESISTANCE:



$$\Delta V = \Delta V_{\max} \sin \omega t.$$

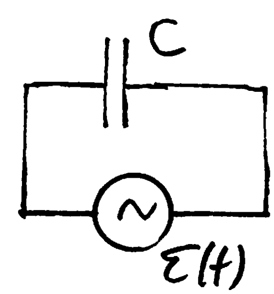
Since $\Delta V = IR$, we get $I(t) = I_{\max} \sin \omega t$. The current "follows" the voltage.

It is useful to define $I_{\text{rms}} = I_{\max}/\sqrt{2}$, and $\Delta V_{\text{rms}} = \Delta V_{\max}/\sqrt{2}$, so that

$$\mathcal{P}_{\text{avg}} = I_{\text{rms}}^2 R.$$

CAPACITANCE:

With capacitor fully charged, or fully discharged, the current is instantaneously zero, so

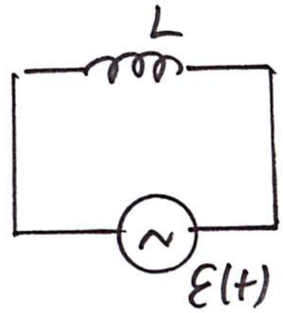


$$I(t) = I_{\max} \cos \omega t.$$

It follows then that

$$\Delta V_{\text{c,rms}} = I_{\text{rms}} X_C \text{ where } X_C = \frac{1}{\omega C}.$$

INDUCTANCE: Since the back-emf of the inductor opposes the change in potential difference, the voltage "leads" the current.

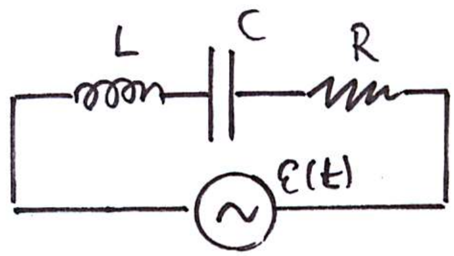


$$I(t) = -I_{\max} \cos \omega t.$$

It follows then that

$$\Delta V_{\text{rms}} = X_L I_{\text{rms}}, \text{ with } X_L = \omega L.$$

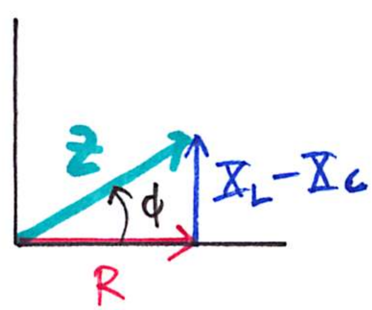
LRC CIRCUIT:



$$\Delta V_{\max} = I_{\max} Z, \text{ where } Z = \sqrt{R^2 + (X_L - X_C)^2}.$$

The angle between $\Delta V_{\max}$ and ΔV_R, or between Z and R, is called ϕ and satisfies

$$\tan\phi = \frac{X_L - X_C}{R}.$$



Power in AC circuits:

$$\mathcal{P}_{\rm avg} = I_{\rm rms}^2 R = I_{\rm rms}\Delta V_{\rm rms}\cos\phi.$$

$$I_{\rm rms} = \frac{\Delta V_{\rm rms}}{Z}.$$

Since Z is a minimum when $X_L = X_C$, then $I_{\rm rms}$ is a maximum under that condition. This is called "resonance."

This condition means $\omega L = 1/(\omega C)$, so that $\omega = \sqrt{1/(LC)}$. Thus the so-called resonant frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

Transformers:

$$\Delta V_1 = -N_1\frac{\Delta\Phi_B}{\Delta t},\ \Delta V_2 = -N_2\frac{\Delta\Phi_B}{\Delta t},$$

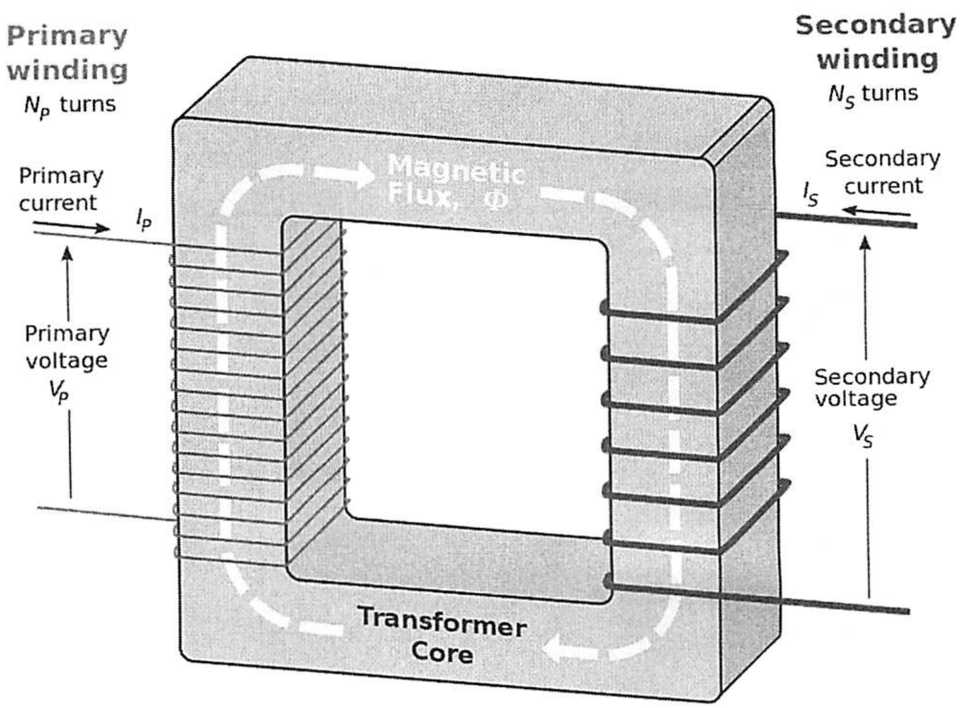
so that

$$\frac{\Delta V_1}{N_1} = \frac{\Delta V_2}{N_2}.$$

Since the power input equals the power output, ignoring resistance losses, then

$$I_1 \Delta V_1 = I_2 \Delta V_2.$$

By varying the windings of the two sides of a transformer one can use it to "step down" or "step up" voltage. Supplying of electricity by utilities would not be possible without transformers... usually there is one on every block of every street in every city!



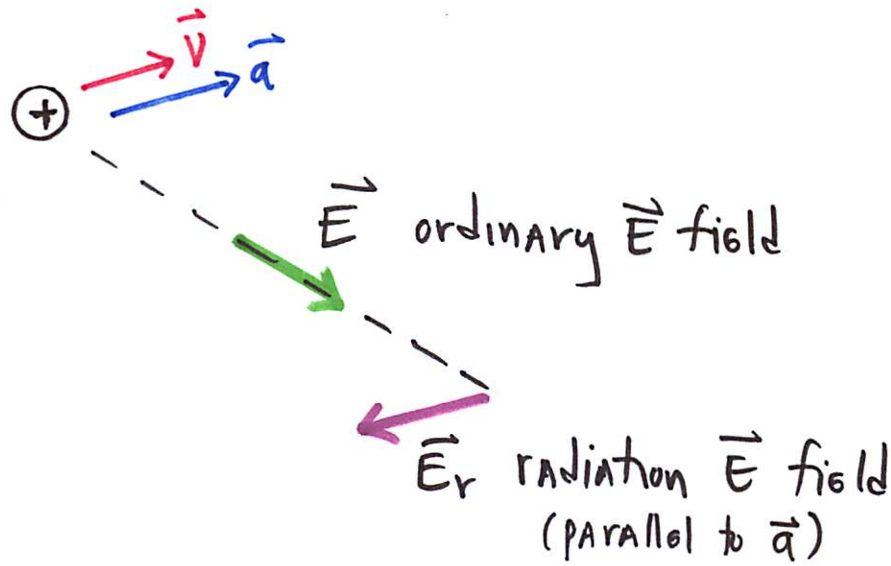
The triumph of 19th Century physics...

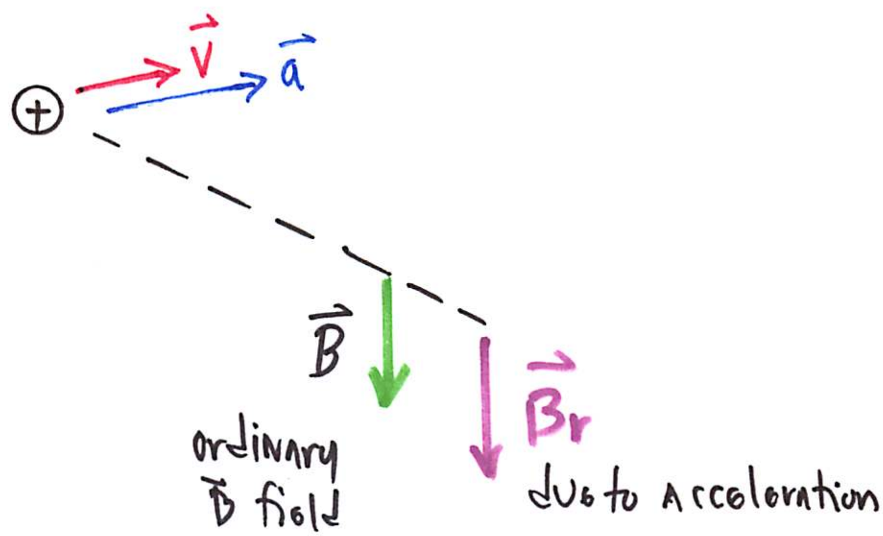
James Clerk Maxwell wrote down four famous equations that summarized everything learned about electric and magnetic fields by the early 1860s. He found that these equations had a solution that predicted a totally new combination of electromagnetic fields, produced when charges *accelerate*. These fields are wavelike and move with a speed $c = (\sqrt{\epsilon_0 \mu_0})^{-1}$, the speed of light! That is $c = \lambda f$.

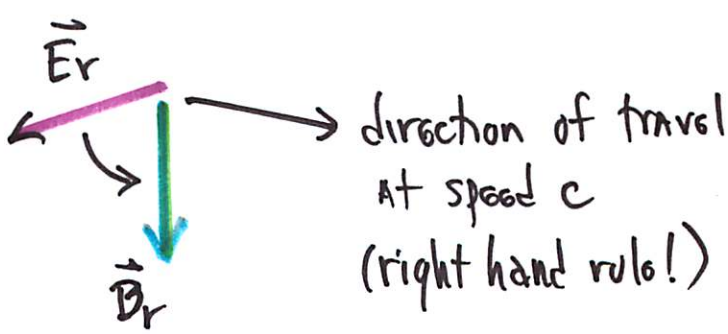
But what were these fields? A transverse propagating disturbance in things made up by Faraday is not a very satisfying physical picture!

In 1905, Einstein showed that the electromagnetic field has to be made of particles, which he called *photons*. By 1930 physicists knew that all fundamental fields are made of particles, called *bosons*. But if the field is made of particles what is "waving'?

In the 1920s Max Born showed that in the new quantum physics, any particle with definite energy and momentum is described by *a wave of probability*. Thus the astonishing result is that electromagnetic waves are "made of probability." We will discuss this more toward the end of the course. It means that where the wave is most intense, you will find most of the photons.


$\vec{v}$
$\vec{a}$
$\vec{E}$ ordinary $\vec{E}$ field
$\vec{E}_r$ radiation $\vec{E}$ field
(parallel to $\vec{a}$)


$\vec{v}$
$\vec{a}$
$\vec{B}$
ordinary
$\vec{B}$ field
$\vec{B}_r$
due to acceleration


$\vec{E}_r$
$\vec{B}_r$
direction of travel
at speed c
(right hand rule!)

Maxwell showed that in an EM wave,

$$\frac{E}{B} = c, \text{ and } I = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0}.$$

He also showed that the wave has momentum. If it has an energy U it has momentum $p = U/c$.

Electromagnetic Spectrum

The range of EM radiation easy to detect has frequencies from 10^{22} Hz to 100 Hz, and corresponding wavelengths from 10^{-14} m to 10^6 m.

Visible light ranges from about 4×10^{-7} to 7×10^{-7} meters, a very narrow range, dictated by the peak radiation output of our star, the sun.

Remember from Ch. 11,

$$\mathcal{P} = \sigma \varepsilon A T^4.$$

The distribution with frequency is such that

$$\lambda_{\text{peak}} T = 2.9 \times 10^{-3} \text{ mK},$$

$$f_{\text{peak}} = 10^{11} T \text{ Hz/K}.$$

PHOTONS:

Electromagnetic radiation consists of particles called (by Einstein) *photons*. Photons have no charge and no mass, but can carry kinetic energy and momentum. If we have a beam of visible light which has frequency f and wavelength λ, the photons have kinetic energy

$$K = hf,$$

where the fundamental constant $h = 6.6 \times 10^{-34}$ J-s or 4.14×10^{-15} eV-s. They have momentum

$$p = h/\lambda = K/c,$$

where c is the speed of light in a vacuum, about 3×10^{8} meters per sec.

The usual unit of momentum used by physicists is the eV/c. A photon with a K of 2.4 eV thus has a momentum of 2.4 eV/c.