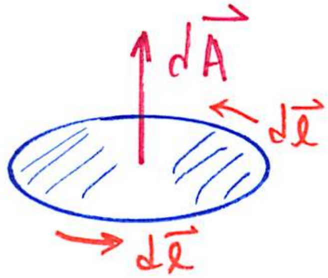
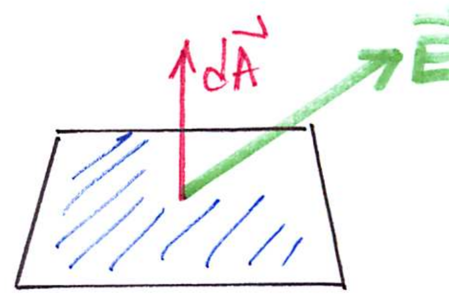
In physics, area is a vector and we can define the differential vector $d\mathbf{A}$ for part of a surface using a right-hand rule.



We can then define the flux of an electric field through a surface element as $d\Phi_E = \mathbf{E} \cdot d\mathbf{A}$.



And then we can define the flux of $\mathbf{E}$ through a closed surface as

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A}.$$

The integral is taken over the entire surface.

The Gauss Law:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inside}}}{\epsilon_0}.$$

Here the constant ϵ_0 is equal to $1/(4\pi k)$.

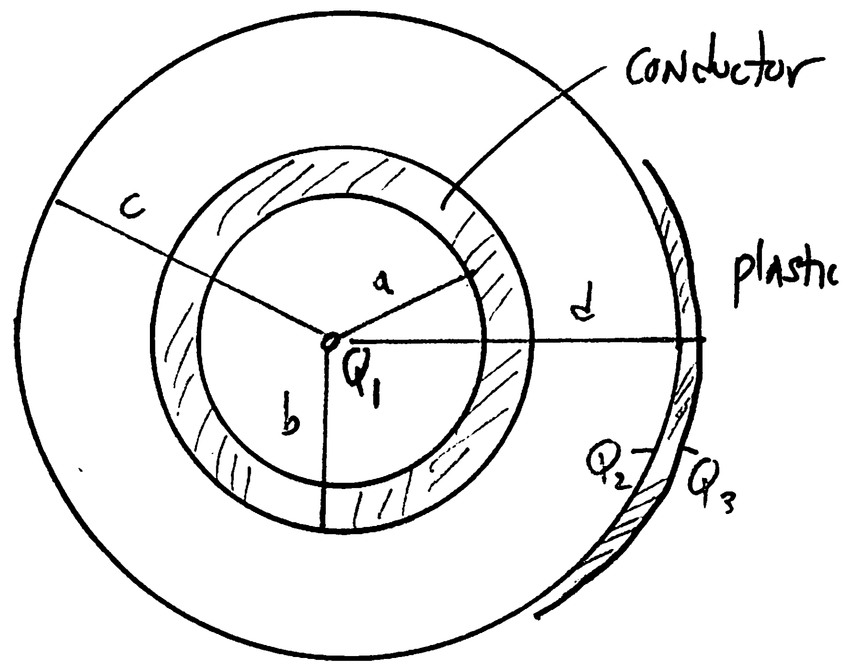
- As a check on the Gauss Law, find the field due to a point charge.

- Now consider a sphere of charge, with the charge per unit volume varying like $\rho(r) = ar$, where a is a constant, and the overall radius of the sphere being R. If the total charge in the sphere is Q, use the Gauss law to show that inside the sphere, $\mathbf{E}(r) = \widehat{\mathbf{r}}Qr^2/(4\pi\epsilon_0 R^4)$. Then use it to show the charge outside is the same as that of a point charge Q. Show the solutions agree at $r = R$.

- Check the Gauss Law again by getting the field of a line of charge, $\mathbf{E}(r) = \lambda\widehat{\mathbf{r}}/(2\pi\epsilon_0 r)$.

- What is the field anywhere above an infinite sheet of charge, with charge σ per unit area? You should find $\mathbf{E}(z) = \widehat{\mathbf{k}}\sigma/(2\epsilon_0)$, where the z axis is perpendicular to the sheet.

- Suppose a point charge Q_1 is surrounded by an initially uncharged conducting shell of inner radius a and outer radius b. This system is in turn surrounded by a plastic shell which bears a charge Q_2 on its inner surface and a charge Q_3 on its outer surface. The shell's inner radius is c and its outer radius is d. Find the field $\mathbf{E}(r)$ everywhere, that is for $r < a$, $a < r < b$, $b < r < c$, $c < r < d$, and $r > d$.



conductor
plastic
c
a
d
b
Q_1
Q_2
Q_3

For $r < a$ the only thing enclosed is the point charge Q_1 so we have according to the Gauss Law, $\mathbf{E} = \hat{\mathbf{r}} k Q_1 / r^2$, just the field of the point charge.

For $a < r < b$ we are inside the conductor so $\mathbf{E} = 0$. A charge of $-Q_1$ is induced on the inner surface of the conductor, so that the field lines from Q_1 stop at the surface.

For $b < r < c$ we are in the empty space between the conductor and plastic shell. A charge of $+Q_1$ must be on the outer surface of the conductor, since it is isolated and must have total charge zero. Therefore the Gauss law gives $E \times 4\pi r^2 = (Q_1 - Q_1 + Q_1)/\epsilon_0 = Q_1/\epsilon_0$. So the field is $\mathbf{E} = \hat{\mathbf{r}} k Q_1 / r^2$.

The plastic shell has a charge of Q_2 on its inner surface and a charge of Q_3 on its outer surface, for a total charge $Q_2 + Q_3$. Therefore inside the plastic shell, $c < r < d$, we get $\mathbf{E} = k\hat{\mathbf{r}}(Q_1 + Q_2)/r^2$.

Finally, outside both shells, $r > d$, we should have $\mathbf{E} = \hat{\mathbf{r}} k (Q_1 + Q_2 + Q_3)/r^2$.

An insulating sphere of radius 5 cm is surrounded by a hollow conducting sphere with an inner radius of 20 cm and an outer radius of 25 cm. The insulator and conductor both have a net charge. The electric field $\mathbf{E}$ at a distance of 10 cm from the center of the system is found to be 3600 N/C radially inward, while the field at a point 50 cm from the center of the system is found to be 200 N/C radially outward. (a) What is the charge of the insulating sphere? (b) What is the net charge on the conducting sphere? (c) What is the charge on the inner surface of the conducting sphere? (d) What is the charge on the outer surface of the conducting sphere?