Ch. 20 Examples:

- Suppose a single loop of wire has a radius of 0.1 m and a resistance of 1 Ohm. If a uniform magnetic field is directed downward perpendicular to the plane of the loop and is increasing in strength at a rate of 10^{-2} T/s, what are the induced emf and current in the loop? What's the direction of the current?

If we apply Faraday's law of induction, since B is the only thing changing with time,

$$\mathcal{E} = -((\Delta B)/\Delta t)\pi r^2.$$

Plugging in the numbers we get something like $-\pi \times 10^4$ Volts. The minus sign just means the induced emf and current oppose the flux change, so that means the current has to circulate in such a direction that the resulting **B** points in the opposite direction to the direction **B** is increasing, namely upward, and that means the induced current must circulate counterclockwise in the loop as seen from above. Its magnitude is of course $|\mathcal{E}|/R = \pi \times 10^{-4}$ Amps.

- As discussed in class, a solid conducting rod is travelling through a uniform magnetic field. If the rod is sliding on a U-shaped conductor, it forms part of a conducting loop of ever-increasing area. What is the induced current at a certain moment, if the B field magnitude is 10^{-4} T, the length of the rod is 1 m, the resistance of the circuit at that certain time is 1 Ohm, and the speed of the rod is 100 m/s?

The flux change is $B\Delta A = B\ell\Delta x$ so the induced emf has magnitude $B\Delta A/\Delta t = B\ell v$. Thus the current is $I = B\ell v/R = 10^{-2}$ Amps.

- What is the self-inductance of a coil if at some instant the total energy stored in the magnetic field within the coil is 100 J, and at that instant the current is 10 Amps?

$U = (1/2)LI^2$ so $L = 2U/I^2$ and if we plug in the numbers we get 2 Henries.

In a series RL circuit, the switch is thrown to put the resistor and inductor across a battery with a voltage of 100 Volts. If the resistance is 1 Ohm and the inductance is 1 Henry, what is the voltage across the inductor after 1 second?

With calculus you can show $I(t) = [\mathcal{E}/R](1 - \exp(-Rt/L))$. Again with calculus and the definition of self-inductance, this result leads to $V_L = \mathcal{E}\exp(-Rt/L)$. So that's the answer. If we plug in the numbers, we get 100 Volts $\times \exp(-1) = 36.8$ Volts.