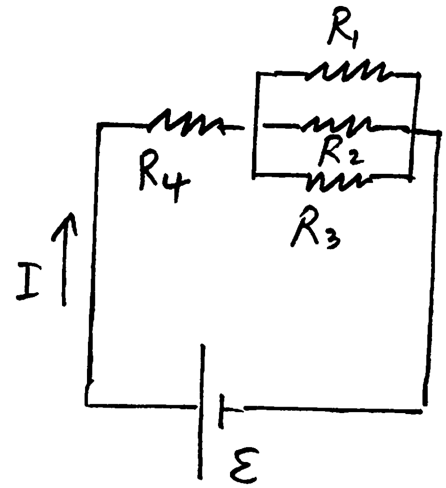


A network of resistors is as shown, with $R_i = i\ \Omega$ and $\mathcal{E} = 12$ V. What is the power lost in the $1\ \Omega$ resistor?

How to proceed?

(1) It's a straightforward network so combine resistances to get the total resistance R_{1234}.

(2) Then use Ohm's law to find the total current I in the circuit.

(3) Then use what you can figure out about individual currents and individual potential drops to get either the current through R_1 or the potential drop across R_1.

(4) Then use whichever one of the three expressions for $\mathcal{P}$ involves the quantities you know.

Answer: 2.07 Watts. A straightforward check is to get the power lost in each resistor and make sure the individual losses add up to the total loss, $\mathcal{P}_{\text{tot}} = I^2 R_{1234}$.

Resistor network question:

We need to find R_{123} which we can easily do: $R_{123} = R_1R_2R_3/(R_2R_3 + R_1R_3 + R_1R_2)$. Plugging the numbers in, we find that the equivalent resistance is about 0.545 Ohms. Since $R_{\rm tot} = R_4 + R_{123} = 4.545$ Ohms we can now get the total current in the circuit, $I = \mathcal{E}/R_{1234}$. This is 2.64 Amps. We will just call $\mathcal{E}$ V as usual.

From the diagram, the potential difference across resistors 1, 2 and 3 is the same (they are in parallel), so $V = V_4 + V_{123}$. But $V_4 = IR_4$ which is 10.56 Volts. And so $V_{123} = V - V_4 = 1.44$ V.

Now $V_{123} = I_1R_1$ so $I_1 = 1.44$ Amps.

So finally $\mathcal{P}_1 = I_1^2R_1 = 2.07$ Watts.

It's laborious, but not conceptually difficult, to check this answer:

$\mathcal{P}_{\rm tot} = I^2R_{\rm tot} = 31.68$ Watts.

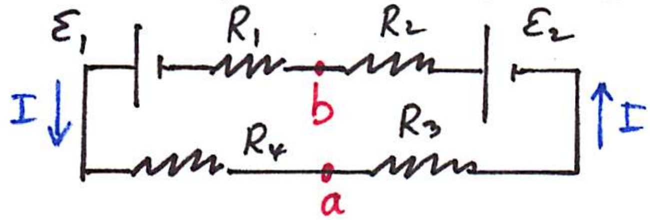
$\mathcal{P}_2 = (V_{123}/R_2)^2R_2 = 1.037$ Watts.

$\mathcal{P}_3 = 0.69$ Watts by the same method.

$\mathcal{P}_4 = I^2R_4 = 27.88$ Watts.

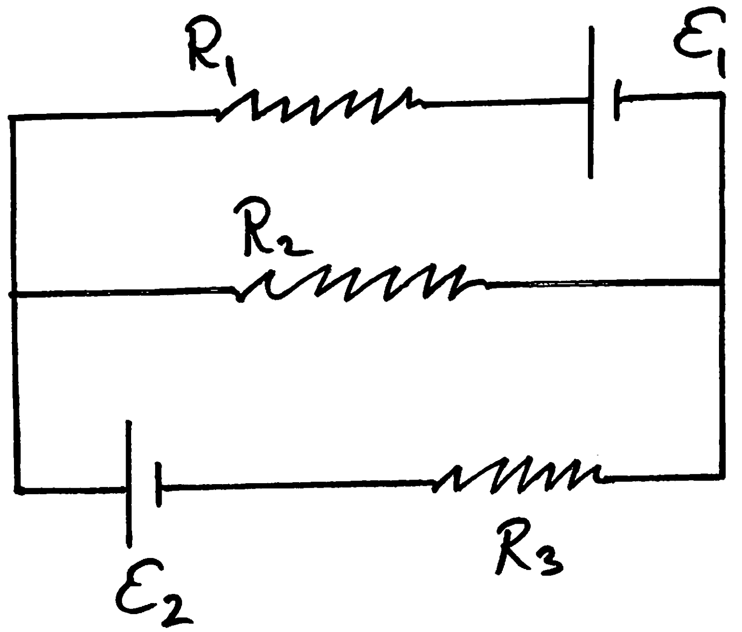
It's now clear that indeed $1.04 + 2.07 + 0.69 + 27.9 = 31.68$.

A simple circuit is sketched as shown. In the circuit, $\mathcal{E}_i = i$ Volts and $R_i = i\ \Omega$. If we assume the current I flows as shown, what is $V_b - V_a$?



Answer: $I = (\mathcal{E}_1 + \mathcal{E}_2)/(R_1 + R_2 + R_3 + R_4)$ and $V_b - IR_1 + \mathcal{E}_1 - IR_4 = V_a$, when we loop counterclockwise from b to a.

Combining results, $V_b - V_a = 0.5$ V.



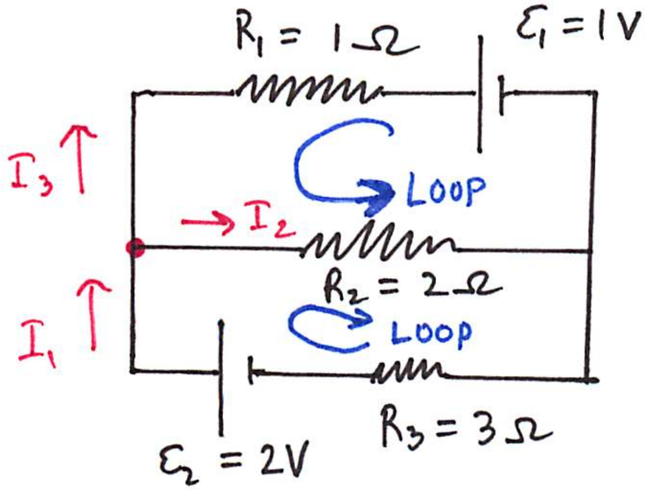
Given R_i and $\mathcal{E}_i$, find currents!

3 unknowns...

5 knowns...

Let $R_i = i\,\Omega$

$\mathcal{E}_i = i\,V$



Junction: $I_1 = I_2 + I_3$

Loop 1: $\mathcal{E}_2 - I_2 R_2 - I_1 R_3 = 0.$

Loop 2: $-\mathcal{E}_1 + I_3 R_1 - I_2 R_2 = 0.$

Also: $\mathcal{E}_1 + \mathcal{E}_2 = I_3 R_1 + I_1 R_3.$

So: $2 = 5I_2 + 3I_3,\ 1 = I_3 - 2I_2,$ or $I_3 = 1 + 2I_2.$

Combine to get: $2 = 11I_2 + 3$ so $I_2 = -(1/11)$ Amps.

Then $I_3 = (9/11)$ Amps.

So $I_1 = (8/11)$ Amps.

To check note $1 + 2 = (9/11) \times 1 + (8/11) \times 3 = 3.$

Two RC Circuit Questions:

(1) In a series RC circuit, a switch is closed and the current builds up, charging the capacitor. How much time does it take to charge the capacitor to 90% of its maximum possible charge? R is 1 Ohm and C is 1 μF.

$$q(t) = Q[1 - \exp(-t/RC)]$$

so $q/Q = 0.9 = 1 - \exp(t/RC)$ which means $\ln(\exp(-t/RC)) = \ln(0.1)$. Remember that $\ln(\exp x)) = x$, since Napier's number e is the base of natural logarithms.

This gives us $t = 2.3RC = 2.3 \times 10^{-6}$ sec.

(2) If the battery is taken out of the circuit and the fully charged capacitor is discharged through the resistor, how long does it take to get down to 10% of its full charge. Assume $C = 2$ μF and $R = 2$ Ohms.

Now $q(t) = Q\exp(-t/RC)$, so we have $\ln(q/Q) = -t/RC$, or $t = -RC\ln(q/Q)$. Plugging in the numbers gives $t = +9.2 \times 10^{-6}$ sec.