

## Worked Examples, Ch. 16:

(1) Imagine a charge in a uniform electric field  $\mathbf{E}$ . Starting from rest, the charge travels through a potential difference of 100 Volts in moving parallel to the field line a distance of 0.01 m. What is the magnitude of the electric field?

Answer: In this course, since we don't know calculus, we can compute work only for constant forces. For example the work done by a constant force  $\mathbf{F}$  on a particle that moves a distance  $\Delta x$  along the  $x$  axis is  $W = F_x \Delta x$ . If we imagine dividing this expression by some unit charge  $q$ , and if we understand what  $\mathbf{E}$  and  $V$  are, we see that  $\Delta V = E \Delta x$ . So  $E = \Delta V / \Delta x = 100 \text{ V} / (0.01 \text{ m}) = 10,000 \text{ V/m}$  or  $\text{N/C}$ .

(2) What is the final speed of an electron that falls from rest across a 1000 V potential difference?

Answer: If we realize  $\mathcal{E} = K + U$ , and  $\Delta \mathcal{E} = 0$ , then if the final speed is much less than the speed of light, we can write  $\Delta K = -\Delta U$  or

$$(1/2)m_e v^2 = eV.$$

So

$$v = \sqrt{\frac{2eV}{m_e}}.$$

Here  $e = 1.6 \times 10^{-19} \text{ C}$  and the charge on the electron,  $q_e = -e$ . And of course  $m_e$  is the electron mass, about  $9.1 \times 10^{-31} \text{ kg}$ . Solving for  $v$  by plugging in the numbers we get  $v = 1.9 \times 10^7 \text{ m/s}$ . Compare to the speed of light,  $c = 3 \times 10^8 \text{ m/s}$ , and we see even a modest voltage gives us a speed that is already 6% of the speed of light!

(3) A conducting sphere of radius  $R$  has a charge  $Q$  deposited on its surface. If we take the usual definition for the zero of potential for a finite system to be that it goes to zero at infinity, a choice built into the point charge result for  $V$ , namely  $V(r) = kQ/r$ , and if we realize from Gauss's law that the  $\mathbf{E}$  field of a spherical conductor of radius  $R$  with charge  $Q$  on its surface is the field of a point charge for any  $r > R$ , then we can take the  $V$  on the surface of a spherical conductor to be  $V = kQ/R$ . This is a capacitor, since it stores charge. What is the capacitance of this system?

Answer: the definition of capacitance is  $C = Q/V = Q/\Delta V$ . [The  $\Delta V$  is used when we have two conductors in parallel, or concentric.] Therefore the capacitance of a charged conducting sphere is

$$C = Q/(kQ/R) = R/k = 4\pi\epsilon_o R.$$

(4) Two identical capacitors,  $C_1$  and  $C_2$ , are put in series, and then put in series with two more of the same capacitors,  $C_3$  and  $C_4$ , that are in parallel. What is the equivalent capacitance of the system if each capacitor has a value of  $1 \mu\text{F}$ ?

Answer: For capacitors in parallel the capacitance adds, while for capacitance in series, the reciprocals add. Therefore  $C_{34} = C_3 + C_4$ , and  $(C_{1234})^{-1} = (C_1)^{-1} + (C_2)^{-1} + (C_{34})^{-1}$ . This can be written as

$$[C_2 C_{34} + C_1 C_{34} + C_1 C_2]/(C_1 C_2 C_{34}).$$

Therefore the answer is

$$C_{\text{eq}} = (C_1 C_2 (C_3 + C_4))/((C_1 + C_2)(C_3 + C_4) + C_1 C_2) = 2/(4+1) = (2/5) \mu\text{F}.$$

(5) Capacitances  $C$  and  $2C$  are in parallel, and put in series with capacitance  $3C$ . If the entire system is put across a potential difference  $V$ , how much energy is stored in  $C$ ?

Answer: let the voltage across  $3C$  be  $V''$  and the voltage across the two parallel capacitors be  $V'$ . Then  $V = V' + V''$ . Also  $C' = C + 2C = 3C$ , and

$$(C_{\text{tot}})^{-1} = (3C)^{-1} + (C')^{-1} = 2/(3C) \text{ so } C_{\text{tot}} = (3/2)C.$$

Since  $C'$  and  $3C$  are the same,  $V' = V''$ . Since  $V_i = Q_i/C_i$ , the same amount of positive charge is stored on both  $3C$  and  $C'$ .

Now there are three different ways to write the energy stored in a capacitor. Here the most useful form looks like  $\mathcal{E} = (1/2)C\Delta V^2$ .

The energy stored in  $C$  is then  $\mathcal{E}_C = (1/2)C(V/2)^2 = CV^2/8$ .

We can check this result by calculating the total energy stored, and the energy stored in  $2C$  and in  $3C$ . The total energy stored is (using the total capacitance)  $(3/4)CV^2$ . The energy stored in  $3C$  is  $(3CV^2)/8$ . And the energy stored in  $2C$  is  $CV^2/4$ . This checks because the sum of the energies for each of the three capacitors does equal the total stored energy.

(6) A capacitor of capacitance  $C$  is hooked to a battery of voltage  $V$ . While the capacitor remains hooked to the battery, a dielectric slab of dielectric constant  $\kappa$  is inserted between the plates of the capacitor. By what factor has the charge on the positive plate changed after the slab is inserted?

Answer: Insertion of the dielectric changes the capacitance from  $C$  to  $\kappa C$ . The voltage remains constant. So before the dielectric is inserted,  $C = Q/V$ , and afterward  $\kappa C = Q'/V$ . If we divide the 2nd equation by the first, we get  $\kappa = Q'/Q$ , so  $Q' = \kappa Q$ . More charge is pulled from the battery after the slab is inserted, to maintain the same potential difference across the capacitor.