• A thin insulating spherical shell of radius b has a total charge $+Q$ distributed uniformly through it. Inside the shell is a solid conducting sphere of radius a, with no surface charge. Find the electric field everywhere in space.

• A thick conducting spherical shell has inner radius a and outer radius b. A total charge Q is dumped on the outer surface. Now a point charge $-5Q$ is placed at the center of the shell. Find the electric field everywhere, and the charge on both surfaces of the shell.

A long cylindrical pipe made of an insulating material has inner radius a and outer radius b, and a uniform charge density ρ (units $\mathrm{C/m^3}$). Find the electric field everywhere in space.

Consider a solid nonconducting sphere of radius R with a uniform charge density ρ throughout, resulting in total charge Q. We know instantly from Gauss's law that the electric field $\mathbf{E}(r > R)$ is the same as that of a point charge Q. But what is the electric field $\mathbf{E}(r < R)$?

Solutions:

Question 1: Imagine a spherical surface with $r < a$. It encloses no net charge so $E(r < a)$ must be zero. [There is no net charge inside a solid conductor.] Now imagine a spherical surface with $a < r < b$. There is no charge on the surface of the conductor, and the space between the conductor and shell is empty, so again no net charge is enclosed, and $E(b > r > a)$ must be zero. Now imagine a spherical surface with $r > b$. There is a net charge $+Q$ on the insulating shell, so that is the total enclosed charge, and the electric field for $r > b$ is the same as the electric field of a point charge of $+Q$.

Question 2: Imagine a spherical surface with $r < a$. It encloses a net charge, $-5Q$. So the field is the field of a point charge of that amount. Now imagine a spherical surface with $a < r < b$. There cannot be an electric field inside the conductor so there must be a charge $+5Q$ induced on the inner surface of the conductor, to generate all electric field lines that go inward to the central point charge. $E(a < r < b) = 0$. But the conductor has no net charge, so in addition to the charge $+Q$ dumped on the outer surface, there is a charge $-5Q$ left behind when charge is pulled to the inner surface. So the net charge on the outer surface is $-4Q$. Therefore the field for $E(r > b)$ is the field of a point charge of $-4Q$.

Question 3: For the long cylindrical pipe, a cylindrical surface inside the hollow, with $r < a$, encloses no charge, so the electric field is zero for $r < a$. Inside the material of the pipe itself, we choose a surface with radius r and arbitrary length ℓ. By symmetry the $\mathbf{E}$ field is perpendicular to the sides of the cylinder, and parallel to the end "lids." Thus if ρ is the charge per unit volume, Gauss's law gives

$$E 2\pi r\ell = (\rho/\epsilon_0)[\pi r^2\ell - \pi a^2\ell] \text{ so } E = \rho[r^2 - a^2]/(2\epsilon_0 r).$$

The same argument for $r > b$ gives

$$E = \rho[b^2 - a^2]/(2\epsilon_0 r).$$

Question 4: For a solid, nonconducting sphere of radius R, with a uniformly distributed charge $Q = (4/3)\pi R^3\rho$, if we take a spherical surface of $r < R$

the enclosed charge is $(r/R)^3Q = (4/3)\pi r^3\rho$. This tells us using the Gauss law that inside the sphere,

$$E4\pi r^2 = (4/3)\pi r^3\rho/\epsilon_0 \text{ so } E = (r\rho)/(3\epsilon_0).$$

We could write this in terms of Q, and in vector notation, as

$$\mathbf{E}(r < R) = (\mathbf{r}Q)/(4\pi\epsilon_0 R^3).$$

Results such as this are easily checked for algebraic errors by noting that the expression for $E(r < R)$ must give the same result as the expression for $E(r > R)$ on the surface $r = R$. [Remember $\widehat{\mathbf{r}} = \mathbf{r}/r$.]