• A thin insulating spherical shell of radius $b$ has a total charge $+Q$ distributed uniformly through it. Inside the shell is a solid conducting sphere of radius $a$, with no surface charge. Find the electric field everywhere in space.

• A thick conducting spherical shell has inner radius $a$ and outer radius $b$. A total charge $Q$ is dumped on the outer surface. Now a point charge $-5Q$ is placed at the center of the shell. Find the electric field everywhere, and the charge on both surfaces of the shell.

A long cylindrical pipe made of an insulating material has inner radius $a$ and outer radius $b$, and a uniform charge density $\rho$ (units C/m$^3$). Find the electric field everywhere in space.

Consider a solid nonconducting sphere of radius $R$ with a uniform charge density $\rho$ throughout, resulting in total charge $Q$. We know instantly from Gauss’s law that the electric field $\mathbf{E}(r > R)$ is the same as that of a point charge $Q$. But what is the electric field $\mathbf{E}(r < R)$?
Solutions:

Question 1: Imagine a spherical surface with \( r < a \). It encloses no net charge so \( E(r < a) \) must be zero. [There is no net charge inside a solid conductor.] Now imagine a spherical surface with \( a < r < b \). There is no charge on the surface of the conductor, and the space between the conductor and shell is empty, so again no net charge is enclosed, and \( E(b > r > a) \) must be zero. Now imagine a spherical surface with \( r > b \). There is a net charge \( +Q \) on the insulating shell, so that is the total enclosed charge, and the electric field for \( r > b \) is the same as the electric field of a point charge of \( +Q \).

Question 2: Imagine a spherical surface with \( r < a \). It encloses a net charge, \(-5Q\). So the field is the field of a point charge of that amount. Now imagine a spherical surface with \( a < r < b \). There cannot be an electric field inside the conductor so there must be a charge \(+5Q\) induced on the inner surface of the conductor, to generate all electric field lines that go inward to the central point charge. \( E(a < r < b) = 0 \). But the conductor has no net charge, so in addition to the charge \(+Q\) dumped on the outer surface, there is a charge \(-5Q\) left behind when charge is pulled to the inner surface. So the net charge on the outer surface is \(-4Q\). Therefore the field for \( E(r > b) \) is the field of a point charge of \(-4Q\).

Question 3: For the long cylindrical pipe, a cylindrical surface inside the hollow, with \( r < a \), encloses no charge, so the electric field is zero for \( r < a \). Inside the material of the pipe itself, we choose a surface with radius \( r \) and arbitrary length \( \ell \). By symmetry the \( E \) field is perpendicular to the sides of the cylinder, and parallel to the end “lids.” Thus if \( \rho \) is the charge per unit volume, Gauss’s law gives

\[
E2\pi r \ell = \left( \rho/\varepsilon_0 \right) [\pi r^2 \ell - \pi a^2 \ell] \quad \text{so} \quad E = \rho[r^2 - a^2]/(2\varepsilon_0 r).
\]

The same argument for \( r > b \) gives

\[
E = \rho[b^2 - a^2]/(2\varepsilon_0 r).
\]

Question 4: For a solid, nonconducting sphere of radius \( R \), with a uniformly distributed charge \( Q = (4/3)\pi R^3 \rho \), if we take a spherical surface of \( r < R \)
the enclosed charge is \((r/R)^3 Q = (4/3)\pi r^3 \rho\). This tells us using the Gauss law that inside the sphere,

\[ E 4\pi r^2 = (4/3)\pi r^3 \rho/\epsilon_0 \text{ so } E = (r\rho)/(3\epsilon_0). \]

We could write this in terms of \(Q\), and in vector notation, as

\[ \mathbf{E}(r < R) = (rQ)/(4\pi \epsilon_0 R^3). \]

Results such as this are easily checked for algebraic errors by noting that the expression for \(E(r < R)\) must give the same result as the expression for \(E(r > R)\) on the surface \(r = R\). [Remember \(\mathbf{\hat{r}} = r/r\).]