

WAVES!

Basic wave parameters: A , f , λ , and v_p .

Waves are formed by coupled oscillators... in other words, a wave is a propagating disturbance in a medium of coupled oscillators. Note $v_p = \lambda/T = \lambda f$.

A wave can be described by any function whatsoever as long as the argument of the function is $x \mp v_p t$.

Transverse Waves: The standard wave function, for example for waves on a string, is

$$y(x, t) = A \cos[kx - \omega t]$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f$. A sin works just as well.

Notice that $v_p = \omega/k = \lambda f$.

The classical wave equation is

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v_p^2} \frac{\partial^2 y}{\partial t^2}.$$

Any function whatsoever will satisfy this equation if its argument is $x \mp v_p t$.

The phase speed of waves depends only on the mechanical properties of the material in which the wave

is propagating. For example for waves on a string,
 $v_p = \sqrt{T/\mu}$.

Motion of the individual oscillators in a wave:

$$v_y = \frac{\partial y}{\partial t}, \quad a_y = \frac{\partial v_y}{\partial t}.$$

Power carried by a wave:

$$\mathcal{P}_{\text{avg}} = \frac{1}{2} \mu \omega^2 A^2 v_p.$$

Intensity of 3-dimensional waves:

$$I = \frac{\mathcal{P}_{\text{avg}}}{4\pi r^2}.$$

Longitudinal Waves:

$$s(x, t) = s_m \cos[kx - \omega t].$$

Typically for sound waves, $v_p = \sqrt{B/\rho} = \sqrt{(\gamma p)/\rho}$.

Sound level in decibels (dB):

$$\beta = [10 \text{ dB}] \log\left(\frac{I}{I_0}\right).$$

Here I_0 is 10^{-12} W/m².

Characteristic Features of all waves:

$$y(x, t) = y_1(x, t) + y_2(x, t) + y_3(x, t) + \dots$$

- Superposition, Interference, Dispersion (group velocity is $v_g = d\omega/dk$), beats ($f_B = |f_1 - f_2|$), reflection.

Fixed end-reflection, phase change. Free end reflection, no phase change.

STANDING WAVES are a consequence of reflection and interference (superposition). The secret to dealing with any displayed standing wave is to count the half wavelengths!

Doppler Effect:

$$f_D = f_S \frac{[1 \pm (v_D/v_p)]}{[1 \mp (v_s/v_p)]}$$