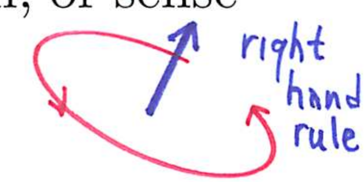
Rigid Body Rotations:

$$\omega = \frac{d\theta}{dt}, \ \alpha = \frac{d\omega}{dt}.$$

Connection to linear quantities: $v = r\omega$, $a_t = r\alpha$, and $a_r = r\omega^2$.

Both $\vec{\omega}$ and $\vec{\alpha}$ are vectors, with direction defined by a right-hand rule based on sense of rotation, or sense of change in $\vec{\omega}$.

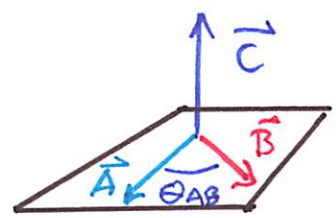


Rigid Rotor Kinematics for constant α:

$$\theta = \theta(0) + \omega(0)t + \frac{1}{2}\alpha t^2.$$

$$\omega = \omega(0) + \alpha t.$$

$$\omega^2 = \omega(0)^2 + 2\alpha(\theta - \theta(0)).$$

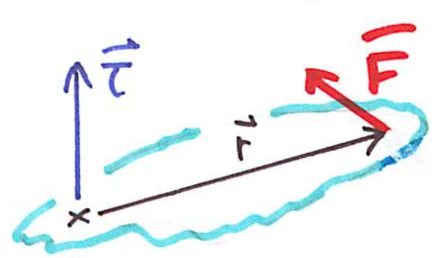


Vector Cross Product: $\mathbf{A} \times \mathbf{B} = \mathbf{C}$.

The vector $\mathbf{C}$ is perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$, and has magnitude $C = AB \sin\theta_{AB}$.

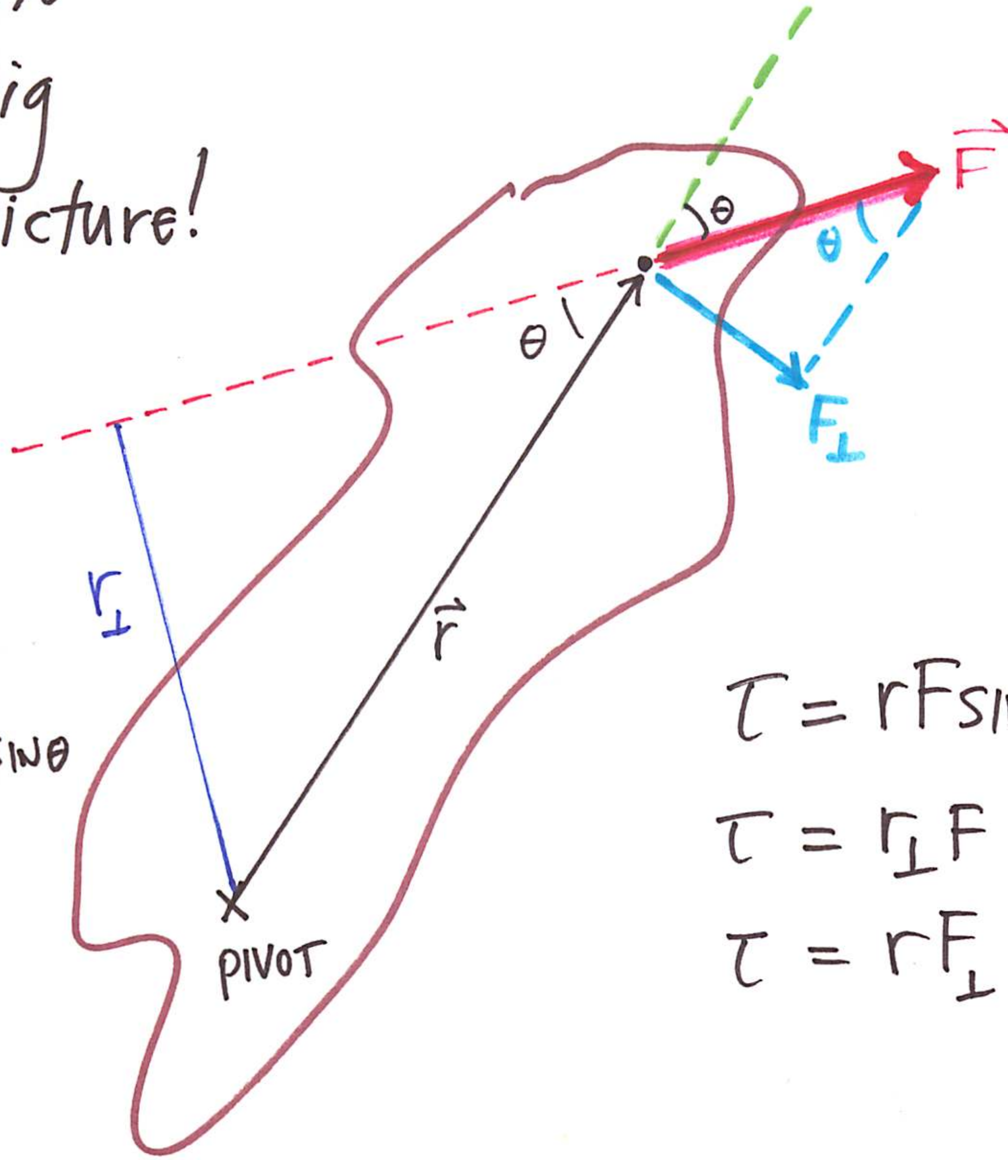
Torque:

$$\vec{\tau} = \mathbf{r} \times \mathbf{F}.$$



$\vec{\tau}$ is $\perp$ to $\vec{r}$ <u>and</u> $\vec{F}$!

The big picture!



$F_\perp = F\sin\theta$

$r_\perp = r\sin\theta$

$\tau = rF\sin\theta$

$\tau = r_\perp F$

$\tau = rF_\perp$

The magnitude of the torque can be expressed three different ways:

$$\tau = rF\sin\theta = r_\perp F = rF_\perp.$$

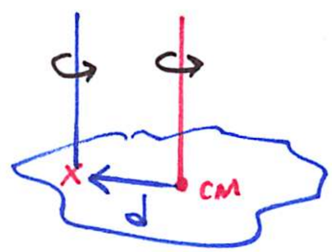
Rotational Inertia:

$$I = \sum_i m_i r_i^2$$

$$I = \int r^2 dm.$$

Parallel Axis Theorem:

$$I = I_{\rm cm} + Md^2.$$



Second Law for Torques!

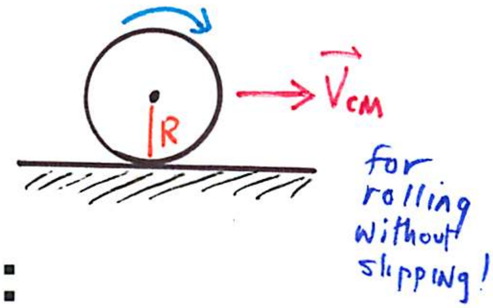
$$\sum_i \vec{\tau}_i = I\vec{\alpha}.$$

Rotational Kinetic Energy:

$$K_r = \frac{1}{2}I\omega^2.$$

Rolling Motion:

$$v_{\text{cm}} = R\omega.$$



Conservation of Energy:

$$E = K_{\text{cm}} + K_r + U_{\text{cm}}.$$