What is \( C_{\text{tot}} \)?
What is \( \Phi_2 \)?
What is \( V_3 \)? (\( V_3 \))

\( C_i = 1 \mu\text{F} \) for all \( i \)
Capacitor network solution:

First $C_{45} = C_4 + C_5$. Then since $C_2$ is in series with that combination, $C_{245} = (C_2C_{45})/(C_2 + C_{45})$. This combination is in parallel with $C_3$ so $C_{2345} = C_3 + C_{245}$. Finally this combination is in series with $C_1$ and the battery of potential $V$. So $C_{12345} = (C_1C_{2345})/(C_1 + C_{2345})$.

Now $C_i$ is 1 $\mu$F for all $i$, so $C_{45} = 2$ $\mu$F, and $C_{245}$ is $2/3$ $\mu$F, so $C_{2345} = 1$ and $2/3$ $\mu$F. Finally $C_{12345}$ is 0.625 or $5/8$ $\mu$F. So this is the equivalent capacitance of the network.

We want to find the charge on $C_2$, and the potential difference across $C_3$.

Looking at the diagram, $V = V_1 + V_3$ and $V_3 = V - Q_1/C_1$ and $Q = Q_1$ where $Q = C_{12345} V$.

Therefore, $V_3 = V (1 - (C_{12345}/C_1))$, which works out to 4.5 V.

Also looking at the diagram, $Q_1 = Q_2 + Q_3$, so $Q_2 = Q - Q_3 = C_{12345} V - C_3 V_3$ which works out to 3 $\mu$C.
A 2 \( \mu F \) capacitor is charged by a 12 V battery and a 3 \( \mu F \) capacitor is charged by a 6 V battery. The capacitors are then removed from the batteries and connected in series so that the + plate of each capacitor is hooked directly to the – plate of the other. When the system comes to equilibrium, what is the + charge on each capacitor, what is the potential drop across each capacitor, and by how much did the total energy stored in the two capacitors change when they were hooked to one another?
Two Capacitors Hooked Together:

Imagine you have two capacitors and two batteries, \( C_1 = 2 \, \mu F \) and \( V_1 = 12 \, V \), \( C_2 = 3 \, \mu F \) and \( V_2 = 6 \, V \). You charge the capacitors to \( Q_1 \) and \( Q_2 \) with the respective batteries. Now you disconnect from the batteries and hook the capacitors together, + plate to − plate. What now are the charges \( q_1 \) and \( q_2 \) on the capacitors?

Well, \( Q_1 = C_1 V_1 \), \( Q_2 = C_2 V_2 \), and by conservation of charge, \( Q_1 - Q_2 = q_1 - q_2 \). Since there is no battery in the new circuit, after the hookup we have to have \( V_1' + V_2' = 0 \). Therefore

\[
\frac{q_1}{C_1} + \frac{q_2}{C_2} = 0.
\]

If we now combine all our results, solving for \( q_1 \) and then \( q_2 \),

\[
q_1 = -q_2(C_1/C_2) = C_1 V_1 - C_2 V_2 + q_2,
\]

so

\[
q_2 = (C_2 V_2 - C_1 V_1)/(1 + (C_1/C_2)) = -3.6 \, \mu C.
\]

Compare to \( Q_2 = C_2 V_2 = 18 \, \mu C \). Therefore \( q_1 = 2.4 \, \mu C \). Compare to \( Q_1 = 24 \, \mu C \).

To check the result, \( Q_1 - Q_2 = 6 \, \mu C = q_1 - q_2 \).

It is also interesting to compare the original stored energy in the capacitors to the final stored energy. You should find, given the numbers we have found, that \( U_i = 2 \times 10^{-4} \, J \), and \( U_f = 3.6 \times 10^{-6} \, J \). Hooking the capacitors + to − nearly discharges them. If they had had equal charge stored, they would have been completely discharged.