(1) \( \mathbf{B} = \hat{i}B \), and \( \mathbf{v} = (\hat{i} + \hat{k})v \) for \( q > 0 \). What is the force exerted on the charge by the magnetic field? \( \mathbf{F} = q\mathbf{v} \times \mathbf{B} \), and \( \hat{i} \times \hat{i} = 0 \), and \( \hat{k} \times \hat{i} = \hat{j} \). So \( \mathbf{F} = \hat{j}qvB \).

(2) If a long straight wire carries current \( I \) and has \( \mathbf{L} = (\hat{i} + \hat{j})L \), in a uniform magnetic field \( \mathbf{B} = \hat{k}B_z + \hat{j}B_y \), what is the force per unit length on the wire? \( \mathbf{F} = I\mathbf{L} \times \mathbf{B} \), so working out the cross products of the unit vectors results in

\[
\mathbf{F}/L = I[(\hat{i} - \hat{j})B_z + \hat{k}B_y].
\]

(3) For the current loop shaped like a letter C on its side, with outer radius \( 2a \) and inner radius \( a \), the magnetic field at the center can be obtained by noticing that \( d\mathbf{l} \times \mathbf{r} \) is zero except on the two arcs. For the inner loop the magnetic field is upward out of the page and if we use

\[
d\mathbf{B} = (\mu_0 I)/(4\pi I)d\mathbf{l} \times \mathbf{r}/r^2,
\]

we get the magnitude

\[
(\mu_0 I\pi a)/(4\pi a^2) = (\mu_0 I)/(4a).
\]
Now for the outer loop the same kind of integration gives a magnitude

\[
(\mu_0 I \pi 2a)/(4\pi (4a^2)) = (\mu_0 I)/(8a).
\]

The vectors are in opposite directions, so the magnitude of the net magnetic field is upward out of the paper and is

\[
(\mu_0 I)/(8a).
\]

(4) If inside a wire carrying net current \( I \) the current density varies as \( j(r) = ar \), and the radius of the wire is \( R \), what is the magnitude of the magnetic field inside the wire, \( r < R \)?

First we need to find \( a \). \( I = \int j \, dA \), which gives

\[
I = 2\pi a R^3 / 3 \quad \text{so} \quad a = 3I/(2\pi R^3).
\]

Now we use Ampere’s Law. Integrating a loop inside the wire gives (using symmetry) \( B2\pi r \). Integrating \( j \cdot dA \) over an area of radius \( r \) inside the wire, and multiplying by \( \mu_0 \), we wind up with

\[
B2\pi r = [2\pi \mu_0 I r^3]/(2\pi R^3).
\]

The final result is that

\[
B(r < R) = \frac{\mu_0 I r^2}{2\pi R^3}.
\]
We can check this result, at once, because for $r = R$ it must agree with the usual result outside a long, straight wire, namely

$$B_{\text{surface}} = \frac{\mu_0 I}{2\pi R}.$$