Simple Biot-Savart Example:

According to this law, which allows you to compute the magnetic field due to a current of any shape,

\[ d\mathbf{B} = \left[ \frac{\mu_0}{4\pi} \right] \frac{d\mathbf{l} \times \mathbf{r}}{r^2}, \]

and this is a hard law to use because for a general current geometry, the angle between \( d\mathbf{l} \) and \( \mathbf{r} \) changes in a very complex way as you integrate along.

As a simple and useful example, consider the field at the center of a circular loop of current \( I \) of radius \( a \). Maybe this is a superconductor which sustains a current of constant magnitude with no resistance loss. Now the integral is very easy because \( \mathbf{r} \) points at the center of the loop and so always makes a right angle with \( d \mathbf{r} \). Therefore the direction of \( \mathbf{B} \) is perpendicular to the plane of the loop, because this plane contains \( \mathbf{r} \) and \( d\mathbf{l} \). If the current is circulating in the loop counterclockwise as seen from above the plane, then \( \mathbf{B} \) is perpendicular to the plane and up out of it toward us as we look down on the plane.

Therefore if the loop is in the \( x-y \) plane,

\[ \mathbf{B} = k \left[ \frac{\mu_0}{4\pi} \right] I \int d\mathbf{l} / a^2, \]
and the integral is just over the length of the loop, $2\pi a$, so the final result is

$$B = k[\mu_0/4\pi](2\pi a)/a^2 = k\mu_0 \frac{I}{2a}.$$ 

We will find this useful.