ISOSPIN:

We can define a generator of rotations in abstract space such that

$$\hat{T}^2|T,T_3\rangle = T(T+1)|T,T_3\rangle,$$

$$\hat{T}_3|T,T_3\rangle = T_3|T,T_3\rangle,$$

and

$$[\hat{T}_i,\hat{T}_j] = i\hat{T}_k.$$

By analogy with spin, $T = 0,\ 1/2,\ 1,\ 3/2$, etc. and of course $-T, -T+1, ... < T_3 < ..., T-1, T$.

Note that $[H_s,\vec{T}] = 0$, since the strong interaction does not couple to charge. However, clearly $[H_{em},\vec{T}]$ is not zero, since charge depends on T_3. Thus the electromagnetic interaction splits the $(2T+1)$ degeneracy that would otherwise exist, and we see **isospin multiplets.**

Example: the nucleon: $|p\rangle$ has $T_3 = +1/2$, while $|n\rangle$ has $T_3 = -1/2$.

Other examples of multiplets: π^+, π^0, and π^-, $T = 1$. Δ^{++}, Δ^+, Δ^0 and Δ^-, $T = 3/2$.

We can generalize the relation between charge and hypercharge to be:

$$Q = e(T_3 + Y/2) = e(T_3 + (1/2)(A+S+C+T+B)).$$

PARITY:

There are two kinds of vectors in physics, polar vectors, and axial vectors. If we define $\mathcal{P}$ so that $\mathcal{P}\mathbf{r} = -\mathbf{r}\mathcal{P}$, etc., then clearly $\mathcal{P}\mathbf{L} = \mathbf{L}\mathcal{P}$.

If $\mathcal{P}|\psi\rangle \propto |\psi\rangle$, then the state is an eigenstate of parity. Define the eigenvalue as η_P. Then clearly $\eta_P^2 = 1$, so the only possible eigenvalues are $\eta_p = \pm 1$.

You remember from 373 that $\mathcal{P}|\ell m\rangle = (-1)^\ell |\ell m\rangle$.

For a reaction $a+b \rightarrow c+d$, we can define the overall parity in each partial wave as

$$\mathcal{P}_i = \eta_P(a)\eta_P(b)(-1)^\ell, \;\; \mathcal{P}_f = \eta_P(c)\eta_P(d)(-1)^{\ell'}.$$

The *intrinsic parity* of a particle can be specified by arbitrarily defining the parity eigenvalues of proton and neutron as +1 and applying conservation of parity to various reactions that in fact conserve parity.

Notation for bound state: J^π, total angular momentum and parity.

Helicity: $h = 2\mathbf{J} \cdot \widehat{\mathbf{p}}/\hbar$.

No particle with mass can be a helicity eigenstate. Also $\mathcal{P}h = -h\mathcal{P}$. Strong and electromagnetic processes conserve parity to a very good approximation, but weak processes violate it.

Charge Conjugation:

If a is any additive quantum number, $\mathcal{C}|a\rangle = |-a\rangle$. In general eigenstates of $\mathcal{C}$ do not exist. The exception occurs when all $a = 0$. Then $\mathcal{C}|0\rangle = \eta_c|0\rangle$ and the "charge parity" $\eta_c = \pm 1$.

Weak Hamiltonians do not appear to commute with $\mathcal{C}$, but strong and electromagnetic Hamiltonians do, to a very good approximation. The problem with weak interactions is that neutrinos appear to be helicity eigenstates.

Time Reversal:

In quantum physics you might think that all time reversal $\mathcal{T}$ would do would be to replace t by $-t$, but this can't be, because

$$H = i\hbar \frac{\partial}{\partial t}.$$

Therefore the $\mathcal{T}$ operation has to take the complex conjugate as well as replacing t by $-t$.

CPT symmetry:

All known processes in nature have Hamiltonians that commute with the combined operation $\mathcal{CPT}$.

However processes have been discovered that have Hamiltonians that do not commute with $\mathcal{CP}$, which means they must also fail to commute with $\mathcal{T}$ in a way that restores $\mathcal{CPT}$ symmetry. Those processes are vitally important in understanding how matter came into existence in the early universe, since it is such processes that treat particles and antiparticles differently and result in a universe consisting entirely of matter, with no antimatter present primordially.

Discrete Symmetries:

$$\text{Parity}: \mathcal{P}\Psi(\mathbf{r},t) = \Psi(-\mathbf{r},t).$$

$$\text{Charge Conjugation}: \mathcal{C}|\mathcal{A}\rangle = |-\mathcal{A}\rangle$$

where $\mathcal{A}$ is the set of all additive quantum numbers for the system.

$$\text{Time Reversal}: \mathcal{T}\Psi(\mathbf{r},t) = \Psi^*(\mathbf{r},-t).$$

It can be shown with essentially no limitations that $[\mathcal{PCT}, H] = 0$ for any conceivable process in nature.

It came as a big shock in 1957 when it was discovered that $\mathcal{P}$ is not a symmetry for weak interactions. However, $\mathcal{PC}$ was "of course" a good symmetry, even for weak processes.

It came as an even bigger shock in 1964 when it was found that for some processes, even $\mathcal{PC}$ was not a good symmetry!

This might have been expected, since clearly we would not have a universe consisting of matter, unless there were processes in the early universe for which $\mathcal{T}$ was not a symmetry. About 1 in every 10^9 processes in the early universe would have had to produce more particles than antiparticles.

To Summarize:

Parity:

$$\mathcal{P}\mathbf{r}\mathcal{P}^{-1} = -\mathbf{r}.$$

$$\mathcal{P}\mathbf{p}\mathcal{P}^{-1} = -\mathbf{p}.$$

$$\mathcal{P}\mathbf{J}\mathcal{P}^{-1} = \mathbf{J}.$$

Charge Conjugation:

$$\mathcal{C}\mathbf{r}\mathcal{C}^{-1} = \mathbf{r}.$$

$$\mathcal{C}\mathbf{p}\mathcal{C}^{-1} = \mathbf{p}.$$

$$\mathcal{C}\mathbf{J}\mathcal{C}^{-1} = \mathbf{J}.$$

$$\mathcal{C}[\text{additive q.n.}]\mathcal{C}^{-1} = -\text{additive q.n.}.$$

Time Reversal:

$$\mathcal{T}\mathbf{r}\mathcal{T}^{-1} = \mathbf{r}.$$

$$\mathcal{T}\mathbf{p}\mathcal{T}^{-1} = -\mathbf{p}.$$

$$\mathcal{T}\mathbf{J}\mathcal{T}^{-1} = -\mathbf{J}.$$

$$\mathcal{T}t\mathcal{T}^{-1} = -t.$$

It is vital to remember that $\mathcal{T}$ *also complex-conjugates!*

Time Reversal on a Free Particle State:

$$\langle \mathbf{r}t|\mathbf{p}E\rangle = \exp[+i(\mathbf{p}\cdot\mathbf{r} - Et)]/\hbar.$$

Remember $\mathbf{p}$ and E are eigenvalues, constants, not operators.

Remember that $\mathcal{T}$ does two things: changes $t \to -t$, and complex conjugates.

Therefore $\mathcal{T}$ operating on $\langle \mathbf{r}t|\mathbf{p}E\rangle$ gives

$$\exp[-i(\mathbf{p}\cdot\mathbf{r} + Et)]/\hbar.$$

We can re-write this as

$$\exp[+i(-\mathbf{p}\cdot\mathbf{r} - Et)]/\hbar,$$

which means the free particle state is converted into a state with the opposite direction of momentum eigenvalue, and nothing else changes.

Thus this behavior is like the effect of classical time reversal on the motion of a free particle.

Neutral K mesons are pseudoscalars, $J^\pi = 0^-$.

$$\mathcal{C}|K^0> = -|\bar{K}^0>, \ \mathcal{C}|\bar{K}^0> = -|K^0> .$$
$$\mathcal{P}|K^0> = -|K^0>, \ \mathcal{P}|\bar{K}^0> = -|\bar{K}^0>, \text{ by convention}$$
$$\text{Thus } \mathcal{CP}|K^0> = +|\bar{K}^0>, \ \mathcal{CP}|\bar{K}^0> = +|K^0> .$$

Thus we form eigenstates of $\mathcal{CP}$ as follows:

$$|K_1^0> = \frac{1}{\sqrt{2}}[|K^0> + |\bar{K}^0>],$$

$$|K_2^0> = \frac{1}{\sqrt{2}}[|K^0> - |\bar{K}^0>].$$

These are the $\mathcal{CP} \rightarrow \pm 1$ eigenstates, respectively.

However, experimentally the states seen are $|K_S>$ and $|K_L>$. The S state decays to two pions ($\mathcal{CP} \rightarrow +1$) with a lifetime of about 10^{-10} sec, while the L state decays to three pions ($\mathcal{CP} \rightarrow -1$) with a lifetime of about 0.5 $\times 10^{-7}$ sec.

But if one looks closely one sees that the L state can decay to 2 pions with a probability of about 10^{-3}.

Thus the observed states must contain small admixtures of the opposite $\mathcal{CP}$ eigenstate. Suppose

$$|K_S> = (1 + |\epsilon|^2)^{-1/2}[|K_1^0> - \epsilon|K_2^0>],$$

$$|K_L> = (1 + |\epsilon|^2)^{-1/2}[\epsilon|K_1^0> + |K_2^0>].$$

Experiment confirmed that in fact the observed decays were due to this kind of mixing, with the individual decays conserving $\mathcal{CP}$. Thus these were called "indirect $\mathcal{CP}$-nonconserving decays." Experimentally $|\epsilon|$ is about 2.2 $\times 10^{-3}$.

Suppose we start with a state at $t = 0$ that looks like:

$$|K^0 >= \frac{1}{\sqrt{2}}[|K_S > + |K_L >].$$

These states have a *different time dependence*, so at $t > 0$ we would have

$$\frac{1}{\sqrt{2}}[a_S(t)|K_S > + a_L(t)|K_L >].$$

If we suppress c and $\hbar$ to make for less typing, since these are decaying states, we can write the time-dependent terms as

$$a_\alpha = \exp[-im_\alpha t]\exp[-\Gamma_\alpha t/2],$$

where of course the average lifetime $\tau_\alpha = 1/\Gamma_\alpha$.

If we define

$$A(t) = (1/2)[a_S(t) + a_L(t)],\ \bar{A}(t) = (1/2)[a_S(t) - a_L(t)],$$

then we can write

$$A(t)|K^0 > + \bar{A}(t)|\bar{K}^0 > .$$

The result is a **damped oscillation**, characteristic of mixing in quantum physics. [A similar but undamped oscillation occurs between the three neutrino flavors, as we will see later.]

$$|A|^2 = (1/4)\big[\exp(-\Gamma_S t) + \exp(-\Gamma_L t)$$
$$+2\exp(-[\Gamma_S + \Gamma_L]t/2)\cos(\Delta m t)\big].$$

$$|\bar{A}|^2 = (1/4)\big[\exp(-\Gamma_S t) + \exp(-\Gamma_L t)$$
$$-2\exp(-[\Gamma_S + \Gamma_L]t/2)\cos(\Delta m t)\big].$$

Here $\Delta m = |m_S - m_L|$.

Table 1. Transformation properties of electric field $\boldsymbol{E}$, magnetic field $\boldsymbol{B}$, charge q, velocity $\boldsymbol{v}$ and force $\boldsymbol{F}$ under charge conjugation (C), parity (P) and time reversal (T).

	C	P	T
$\boldsymbol{E}$	$-\boldsymbol{E}$	$-\boldsymbol{E}$	$\boldsymbol{E}$
$\boldsymbol{B}$	$-\boldsymbol{B}$	$\boldsymbol{B}$	$-\boldsymbol{B}$
q	$-q$	q	q
$\boldsymbol{v}$	$\boldsymbol{v}$	$-\boldsymbol{v}$	$-\boldsymbol{v}$
$\boldsymbol{F}$	$\boldsymbol{F}$	$-\boldsymbol{F}$	$\boldsymbol{F}$

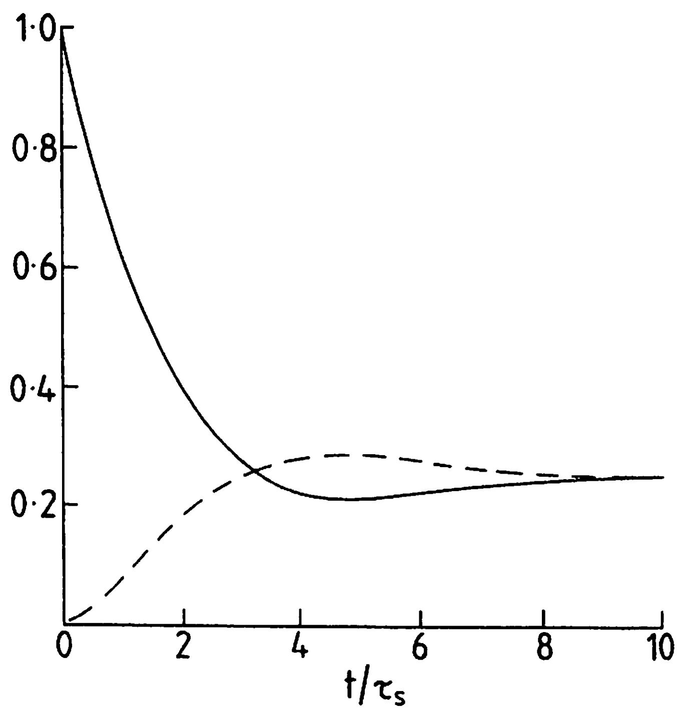

Fig. 9.13 Predicted variation with time of the intensities $I(K^0)$ (solid line) and $I(\bar{K}^0)$ (dashed line) for an initial K^0 beam. The curves are calculated using (9.46) for $\Delta m \cdot \tau_S = 0.5$, where Δm is the mass difference (9.47) and τ_S is the K-short lifetime.