Range of Kinetic Energies used in Nuclear and Particle Physics:

Nuclear: $10^{-3} m_p c^2 < \mathrm{KE} < 10^2 m_p c^2$.

Particle: $m_p c^2 < \mathrm{KE} < 10^4 m_p c^2$.

Note that particle physics is always done at relativistic energies, so that we need to use a Lorentz covariant description from the beginning, not just for kinematics, but also for dynamics.

A "manifestly covariant" approach uses Lorentz scalars ("world scalars") and four-vectors only, if possible.

Examples of world-scalars: mass, charge, other intrinsic particle properties, and fundamental physical constants such as $\hbar$ and c.

Typical four-vector notation:

$$x_\mu = (ct,\ \mathbf{r}),\ x^\mu = (ct,\ -\mathbf{r}).$$

$$p_\mu = ((E/c),\ \mathbf{p}),\ p^\mu = ((E/c),\ -\mathbf{p}).$$

Example of use:

$$-i(p^\mu x_\mu)/\hbar = +i[\mathbf{p} \cdot \mathbf{r} - Et]/\hbar.$$

Dynamical Equations for Relativistic Systems?

- **Klein-Gordon equation.** Describes a single free point particle of mass m with spin zero (only known example, Higgs Boson). To put an interaction into H, you need a covariant description, which is generally not available.

- **Dirac Equation.** Describes a single free point particle of mass m with spin 1/2 (examples, leptons and quarks). To put an interaction into H, you need a covariant description, which is generally not available.

- **Proca Equation.** Describes a single free point particle of mass m with spin 1 (examples, $W^{\pm}$, Z^0). To put an interaction into H, you need a covariant description, which is generally not available.

Really big problem: No form of such equations that describes two interacting particles exists. [Even the classical relativistic two body (Kepler) problem cannot be solved.]

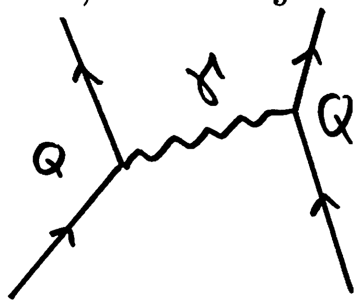
One solution: The Feynman approach: particles are treated as free. Interactions occur via virtual boson emission or absorption. Thus the ingredients of the approach are free particle states, and propagators (Green operators) for the virtual bosons. This approach contains singularities which cannot be eliminated unless the virtual bosons are massless, spin-1 particles. The singularities are due to the point nature of the particles and interactions, and their effect on the vacuum. They can be "concealed" and do not affect the answers, only for "gauge fields" (spin-1 virtual bosons).

Another solution: Avoid the Hamiltonian framework and quantize classical field theory. As a result both particles and fields are treated on precisely the same footing, and the formalism generalizes to any number of mutually interacting fields. This approach contains singularities that have the same cause as those in Feynman's approach, and also include another singularity related to the fact that the fields have a zero-point energy so that when the basic quantity, the Lagrangian density, is integrated over all space, another infinity results. The method of elminating this infinity involves assumptions that have never been confirmed experimentally.

String Theories: get rid of all known infinities, but have never been supported experimentally in any way.

That's why in courses like *Subatomic Physics* you will see very little done with dynamical equations; there is nothing that can play the role played by the Schrödinger Equation in non-relativistic quantum physics.

In the Standard model all forces between fermions are due to exchange of "vector" (spin 1) bosons. The bosons are massless unless gauge symmetry is broken by the Higgs field (we'll explain all these ideas as the course goes on, this is just the "big picture").



$\mathcal{P} \propto Q^2$

Qualitative view:

$$\Delta E \Delta t \simeq \hbar, \;\; \Delta p \Delta r \simeq \hbar.$$

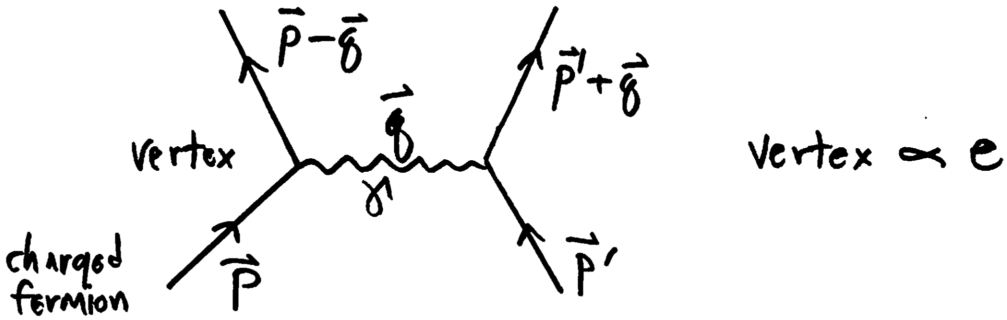
But

$$\Delta E = \Delta p c, \;\text{ and } r \simeq c\Delta t.$$

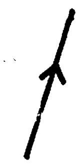
Therefore if $\mathcal{P} = k_e Q^2/(\hbar c)$ then

$$F = \mathcal{P}\frac{\Delta p}{\Delta t} = \frac{k_e Q^2}{r^2}.$$

Feynman Diagrams

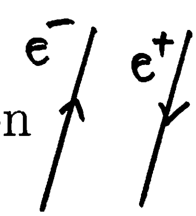


The fermion lines represent plane waves, $\exp(-ip_\mu x^\mu)$, while the exchanged boson is described by a "propagator" which looks like



$$\frac{1}{Q^2 - (mc)^2}, \text{ with } Q^2 = E_Q^2 - \mathbf{q} \cdot \mathbf{q}.$$

This is the operator form of a Green's function.

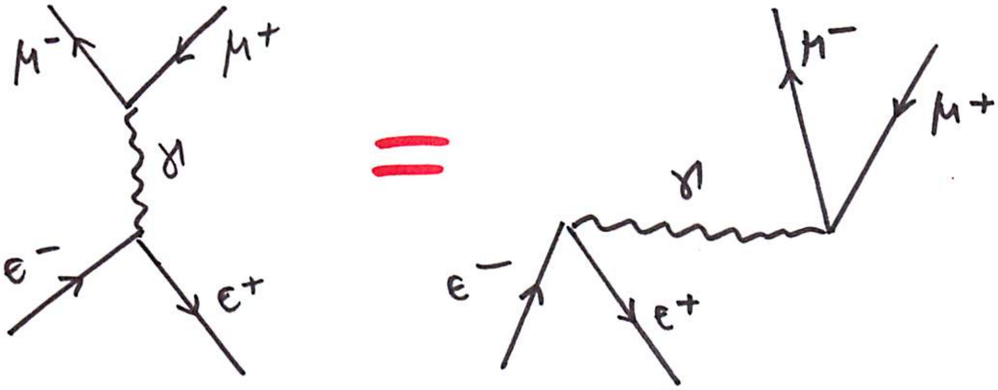


Antifermion lines are reversed compared to fermion lines.

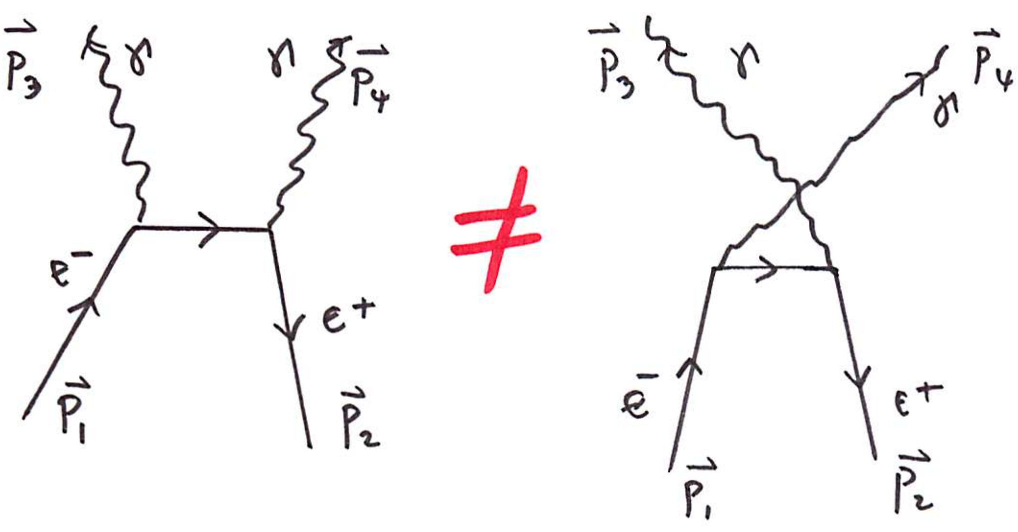
A set of rules allows one to transcribe any Feynman diagram into a matrix element (integral) to be evaluated to obtain the contribution of that type of diagram to the overall amplitude for the process, M_{fi}. If the coupling constant is small enough, the sum over diagram orders rapidly converges.

Feynman Diagram Examples:

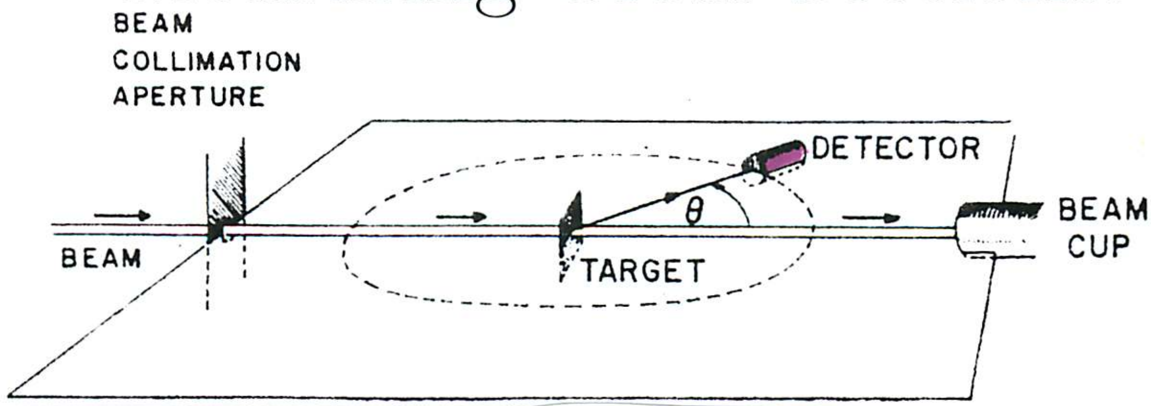
Consider $e^- + e^+ \rightarrow \gamma \rightarrow \mu^- + \mu^+$. Two diagrams that are the same, so that only one needs to be included:



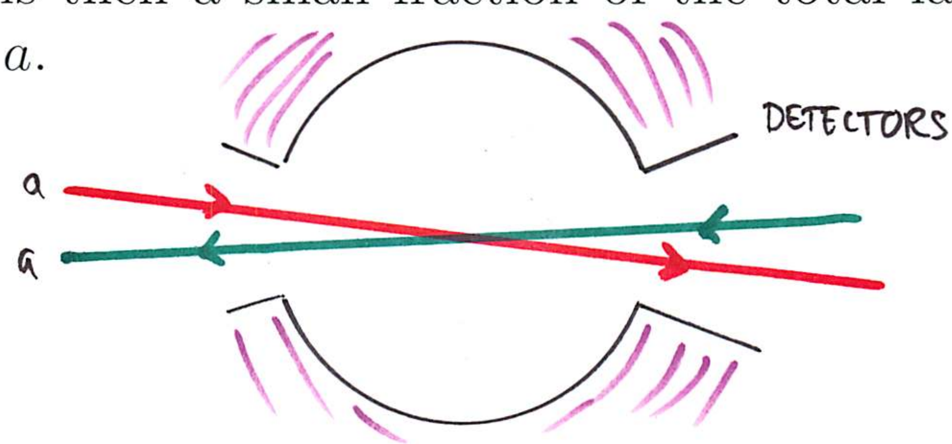
Two diagrams that are different, both must be included, for the process $e^- + e^+ \rightarrow 2\gamma$.



Measuring Cross Sections:



Consider $a + b$ with a the beam particle and b confined to a fixed target. We normally measure $d\sigma/d\Omega$ where $d\Omega$ is the solid angle subtended by the detector, $d\Omega = dA/r^2$. The total cross section is of course $\sigma = \int (d\sigma/d\Omega) d\Omega$. For high beam energies this setup is impractical since the total center-of-momentum energy is then a small fraction of the total lab energy of a.



The solution is colliding beams, which for practical purposes have to be identical, $a + a$, since the two colliding beams are originally a single beam from an injector. In this case the detectors must surround the collision region, and so only the total cross section σ is practical to measure.

Differential Cross Sections aka "Angular Distributions"

Beam flux, # particles per unit area per unit time: $F = n'v$.

n' is beam density in $\#/\text{cm}^3$ and v is particle speed in cm/s.

Scattering centers encountered: $N = nad$.

$n = N_A\rho/A$, # target systems per unit volume, a is the beam area, and d is the target thickness.

Rate of events: $dR_s = FNd\sigma$, so that $R_s = FN\sigma$.

Note that the unit of σ is area, and the unit of $d\sigma/d\Omega$ is area per steradian.

The standard unit used in nuclear & particle physics is the millibarn, mb, which is 10^{-27} cm^2 or 0.1 fm^2. The unit of differential cross section is thus a mb/str.

For Colliders:

$R_i = \sigma\mathcal{L}$ where luminosity $\mathcal{L} = (N_1N_2f)/A_b$. Here f is the frequency of intersection of bunches. Note the units are $(\#\#)/(\text{s-cm}^2) = (\text{cm}^2\text{-sec})^{-1}$.

A typical luminosity for the LHC is 10^{34} cm^{-2}s^{-1}.

Particle Properties:

$$\text{Intrinsic Spin}: \hat{S}^2|sm_s\rangle = s(s+1)\hbar^2|sm_s\rangle.$$

Integer s: boson, number not conserved, no exclusion principle.

Half-integer s: fermion, number conserved, exclusion principle.

Fundamental pointlike fermions:

Leptons: e^-, μ^-, τ^- and antiparticles.

ν_e, ν_μ and ν_τ and maybe antiparticles?

Quarks: u, d, s, c, t and b, and antiparticles.

} Spin $\frac{1}{2}$

Note there are 12 in all.

Fundamental pointlike (Gauge) bosons:

γ, $W^\pm$, Z^0, and 8 color gluons. Spin 1.

Note there are 12 in all.

Fundamental pointline scalar boson:

H^0. Spin 0.

Particle Properties:

Mass:

The mass of fundamental pointlike particles comes from interaction with the Higgs field, which permeates all space.

The mass of composite particles, such as p, n, π^0, etc., comes almost entirely from internal energy... virtual quarks and gluons.

A particle with a finite average proper-frame lifetime, τ, does not have a definite mass because

$$\Delta M c^2 \tau \simeq \hbar.$$

See Sec. 5.3 for methods of mass measurement.

Charge:

Charge is an exactly conserved quantity, and is a Gauge vertex coupling constant.

Charges are found to be Q $= 0$, $\pm(1/3)e$, $\pm(2/3)e$, $\pm e$, etc., where e is the magnitude of the electron charge.

Chargelike quantities which are also Gauge vertex coupling constants include "weak charge," and color.