There are many examples of spontaneous symmetry breaking in physics.

Examples: • Spontaneous magnetization of a ferromagnet cooled below the critical (Curie) temperature. The Goldstone boson is called a *magnon*, a type of "spin wave."

• Buckling of a rod. After the symmetry-breaking buckle, there is a spinning mode corresponding to the Goldstone boson, and oscillations along the length of the buckled rod, modes that require energy to excite.

Electroweak Theory

The unified theory of weak and electromagnetic interactions is a great triumph testifying to the genius of the physicists who created it. It is also insanely complex, a fact that is the fault of nature, not of mankind.

One key to unification is to mimic the apparatus of strong and electromagnetic theories as closely as possible. Thus the introduction of weak isospin, weak charge, weak hypercharge, etc. The other key is to start with a gauge field and break the gauge symmetry at the end.

In the gauge theory, we have bosons with weak isospin $T = 1$ and $T = 0$. The triplet consists of bosons $W^{\pm}$, W^0. The singlet is called B^0.

A parameter called the "Weinberg angle" is introduced to create observed particles γ and Z^0 as mixtures of W^0 and B^0. This is the source of the often-seen (and weird-sounding) remark that the photon is a mixture of both $T = 0$ and $T = 1$, either in terms of weak isospin, or of ordinary isospin.

The usual mixing looks like

$$\gamma = \cos\theta_W B^0 + \sin\theta_W W^0,$$
$$Z^0 = \cos\theta_W W^0 - \sin\theta_W B^0.$$

Since the weak interaction occurs only for left-handed chirality states, we form weak-isospin doublets, $T = 1/2,\ T_3 = \pm 1/2$,

$$\begin{pmatrix} \nu_{eL} \\ e_L^- \end{pmatrix}$$

Weak hypercharge is defined something like

$$q = T_{3\ell} + Y_\ell/2.$$

Looking for example at the doublet above we see $0 = 1/2 + Y_\ell/2,\ -1 = -1/2 + Y_\ell/2$, which gives us in this case $Y_\ell = -1$.

We form similar doublets

$$\begin{pmatrix} \nu_{\mu L} \\ \mu_L^- \end{pmatrix}, \text{ and } \begin{pmatrix} \nu_{\tau L} \\ \tau_L^- \end{pmatrix}.$$

Table 12.1 Multiplets of the electroweak interaction. The quarks d′, s′ and b′ emerge from the mass eigenstates through a generalised Cabibbo rotation (CKM matrix). Weak isospin T doublets are joined in parentheses. The electric charges of the two states of each doublet always differ by one unit. The sign of the third component T_3 is defined so that the difference $z_f - T_3$ is constant within each doublet

	Fermion multiplets			T	T_3	z_f (Q/e)
Leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1/2	+1/2 −1/2	0 −1
	e_R	μ_R	τ_R	0	0	−1
Quarks	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	1/2	+1/2 −1/2	+2/3 −1/3
	u_R	c_R	t_R	0	0	+2/3
	d_R	s_R	b_R	0	0	−1/3

We have talked about weak charge, g or g_w, but in fact there are two weak charges, usually called g and g'. Strictly speaking, g is the coupling to the weak isospin triplet, while g' is the coupling to the singlet. If you require that the photon couples to charged leptons, but not to neutrinos, the result is

$$\tan\theta_W = \frac{g'}{g}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}},$$

and

$$e = g\sin\theta_W.$$

There are many different experiments which involve the Weinberg angle, and a consistent value emerges, namely

$$\sin^2\theta_W = 0.231.$$

Note that this gives $g = e/\sqrt{0.231} = e/0.48$, and thus as we have said earlier that $\alpha_w \simeq 4.3\alpha$.

For electromagnetic processes, then,

$$\begin{pmatrix} 0 \\ e^-_R \end{pmatrix}, \begin{pmatrix} 0 \\ \mu^-_R \end{pmatrix}, \text{ and } \begin{pmatrix} 0 \\ \tau^-_R \end{pmatrix}$$

contribute. As for quarks, weak processes involve only

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \text{ and } \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

while strong processes of course also involve

$$\begin{pmatrix} u \\ d \end{pmatrix}_R, \begin{pmatrix} c \\ s \end{pmatrix}_R, \text{ and } \begin{pmatrix} t \\ b \end{pmatrix}_R.$$

It is possible to predict the masses of the weak bosons in various ways, for example by

$$G_F/\sqrt{2} = \frac{\pi\alpha g^2}{2e^2} \frac{(\hbar c)^3}{(M_W c^2)^2}$$

and

$$\frac{M_W}{M_Z} = \cos\theta_W \simeq 0.88.$$

Agreement with experiment is good. [Remember that in the W boson energy range, α is about $1/128$ and $\sin^2\theta_W$ is 0.231.]

Searches for the Higgs:

Early searches at the LEP (electron-positron collider) tried to produce a Higgs directly via $e^- + e^+ \to Z \to Z + H$, with the Higgs decaying into $b + \bar{b}$ or $\tau^+\tau^-$. No events were seen up to a center of momentum energy of 114 GeV.

Early searches at the TeVatron (proton-antiproton collider) looked for similar reactions. No events were seen up to a center of momentum energy of 180 GeV.

Success came at the LHC in two detectors, ATLAS and CMS. The process here is $p + p$ collisions. The Higgs was clearly seen as a resonance in decay channels such as $\gamma + \gamma$. The mass of the Higgs turns out to be 125 GeV.

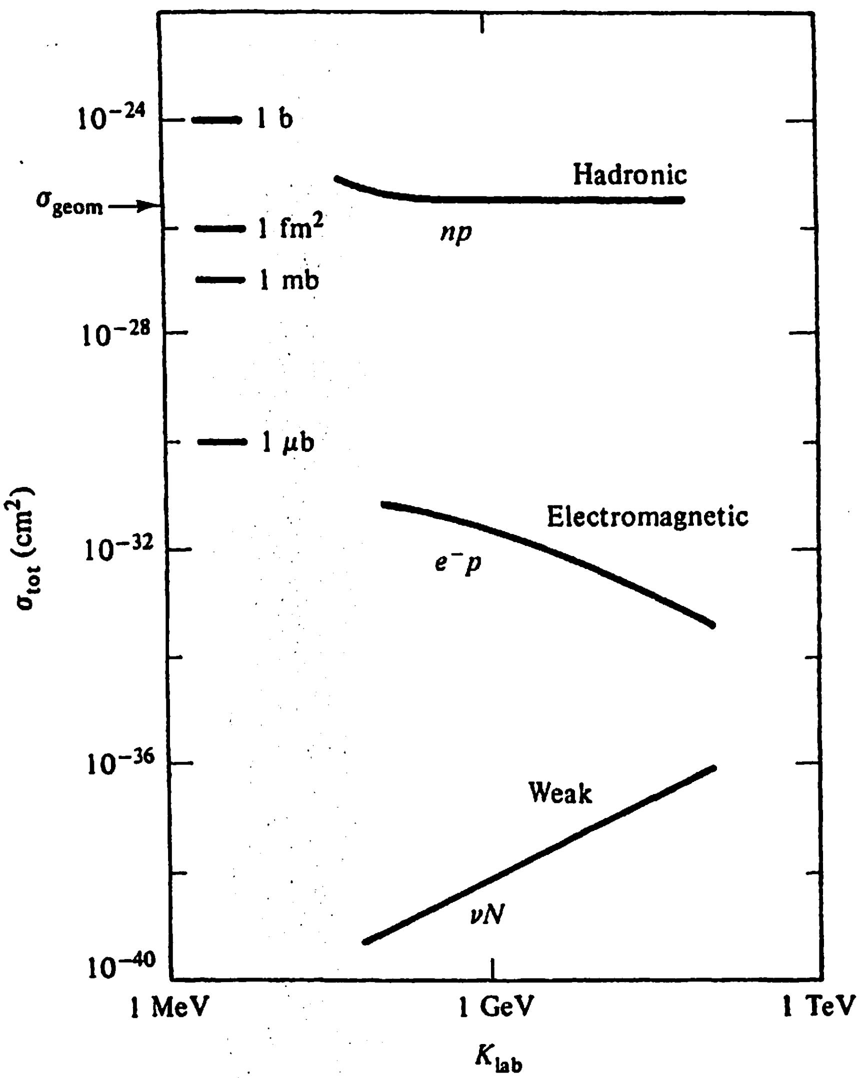

Figure 14.2: Comparison of the total cross sections for strong, electromagnetic, and weak processes on nucleons. σ_{geom} indicates the geometrical cross section of a nucleon, and K is the kinetic energy.

The Strong Nuclear Force:

Beginning in 1932, the study of the atomic nucleus provided physicists with an example of a new force of nature, which did not distinguish between protons and neutrons, but had an extremely short range of around 1.4 fm. In the nucleus, this "strong" force is scarcely 3 times stronger than the electromagnetic repulsion, also.

Early studies of the nuclear force involved $p+p$ and $p+n$ scattering, later supplemented by $\pi+p$ studies.

Results:

- Individual nucleons are complex systems with a radius of about 0.8 fm.
- The nuclear force is weakly attractive at a distance, but becomes strongly repulsive at shorter ranges (easy to understand from the Pauli principle).
- Isospin is a symmetry; the force is charge-independent.

But, alas, the nuclear potential energy depends on just about everything else it could possibly depend upon! In particular the potential has strong spin-orbit, spin-spin, and "tensor" terms, as well as exchange terms.

The nuclear force as a meson field:

Following in the footsteps of Yukawa (1935), there was some success in treating the nuclear force as due to exchange of mesons, particularly the π mesons. Generally phenomenological potentials were constructed, for example of the Yukawa form, with adjustable parameters, to fit the $p + p$ and $p + n$ data.

In the 1970s, a new effort was undertaken to describe the nuclear force in terms of the breaking of chiral symmetry, with the mesons being Nambu-Goldstone bosons. This leads to various effective field theories of the nuclear force.

The deuteron as a test bed

The nucleon-nucleon system has only ONE bound state, and it is just barely bound, at -2.2 MeV. This is the $p + n$ system known as the deutron. It has $J^\pi = 1^+$. It also has isospin zero. The ground state turns out to be a mixture of $\ell = 0$ and 2, due to the so-called "tensor term" in the potential.

Phenomenology:

As a result of the complexity of the potential, the usual approach to the nucleon-nucleon force is a purely phenomenological potential, whose parameters are adjusted to fit all available data. There are a large number of such potentials, developed since the 1950s, and a theorist can pick and choose his favorite.

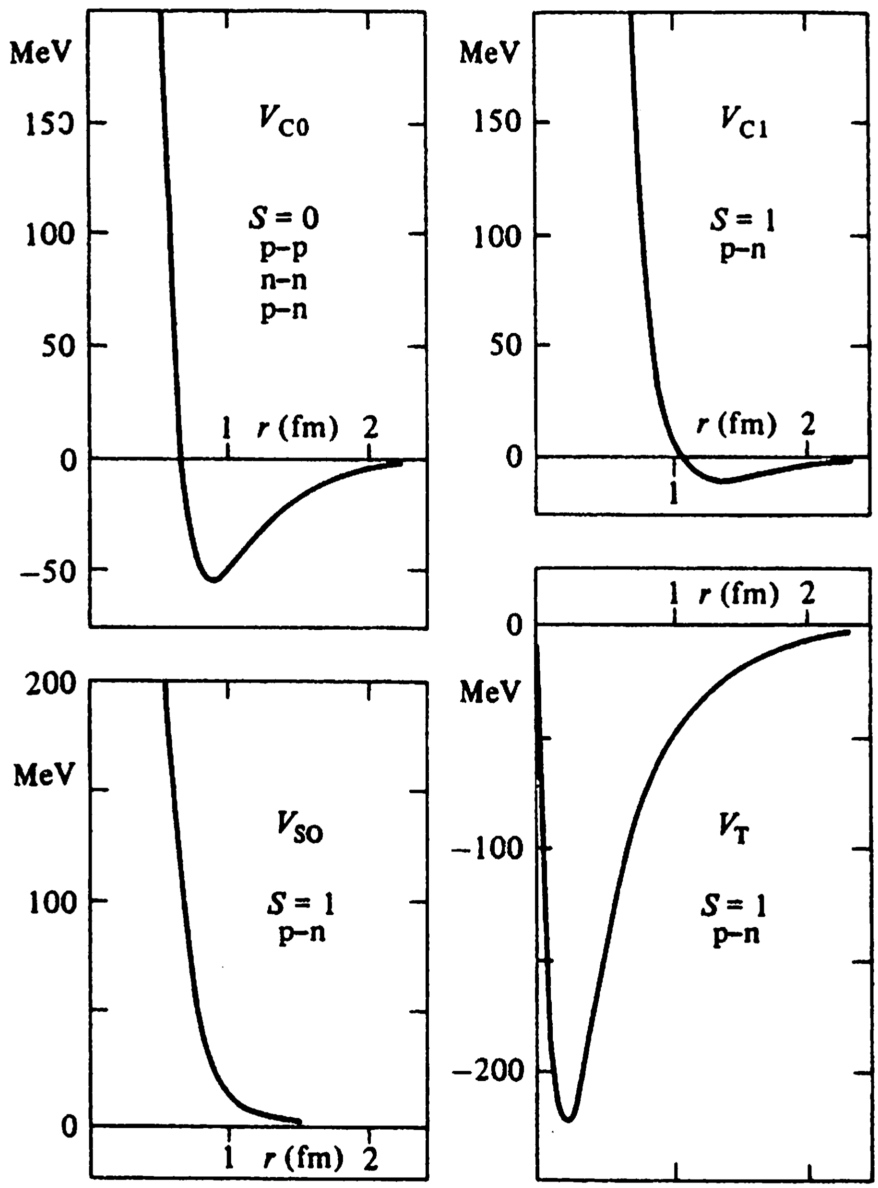

Fig. 3.2 The most important components of the 'Paris potential'. (After Lacombe, M. *et al.* (1980), *Phys. Rev.* **C21**, 861.)

Two expressions are needed: one for the (anti-symmetric) states allowed for two protons or two neutrons, as well as a proton and a neutron, and one for symmetric states accessible only to the neutron–proton system. For both cases, when the spins of the two nucleons are coupled to give a total spin $S = 0$ (see Appendix C) the nucleons only experience a central potential.

In Fig. 3.2, the central potential for the anti-symmetric states with $S = 0$ is denoted by V_{C0}. The central potential for symmetric states differs from this, and is not shown.

When the spins couple to $S = 1$ there are four contributions to these potentials, which are then each of the form

$$V(r) = V_{C1}(r) + V_T(r)\Omega_T + V_{SO}(r)\Omega_{SO} + V_{SO2}(r)\Omega_{SO2},$$

where

$$\begin{aligned}
\Omega_T &= 3\frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{r})(\boldsymbol{\sigma}_2 \cdot \mathbf{r})}{r^2} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\
\hbar\Omega_{SO} &= (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{L} \\
\hbar^2\Omega_{SO2} &= (\boldsymbol{\sigma}_1 \cdot \mathbf{L})(\boldsymbol{\sigma}_2 \cdot \mathbf{L}) + (\boldsymbol{\sigma}_2 \cdot \mathbf{L})(\boldsymbol{\sigma}_1 \cdot \mathbf{L}).
\end{aligned} \tag{3.4}$$

In these expressions $\boldsymbol{\sigma}(\hbar/2)$ is the nucleon spin operator, $\mathbf{L}$ is the orbital angular momentum operator of the nucleon pair, and the subscripts 1 and 2 refer to the two nucleons present.

The fundamental coupling in the strong interaction is to color. There are three colors, $\mathbf{r}$, $\mathbf{g}$ and $\mathbf{b}$. The quarks carry color, so each quark comes in three possible states, say $u_{\mathbf{r}}$, $u_{\mathbf{g}}$ and $u_{\mathbf{b}}$.

The situation with respect to gluons is more complex. The gluons carry color plus anticolor. Based on the meson octets of the old "Eightfold Way," the standard way to classify gluons is into an octet and a singlet. The singlet state is symmetric in color space and does not correspond to a gluon. So the eight possible gluons might be written (there are of course other choices) as

$$r\bar{g},\ r\bar{b},\ g\bar{r},\ g\bar{b},\ b\bar{r},\ b\bar{g},\ \text{and}$$

$$\frac{1}{\sqrt{2}}[r\bar{r} - g\bar{g}] \text{ and } \frac{1}{\sqrt{6}}[r\bar{r} + g\bar{g} - 2b\bar{b}].$$

We might expect the potential between quarks to be hydrogen-like, and it is at short distances, but at larger distances, the force becomes constant, so that the actual potential looks pretty much like

$$V(r) = \frac{-\alpha_s k}{r} + Ar,$$

and by fitting meson excited states it's possible to determine the potential parameters experimentally to a satisfactory degree.

- Color Isospin and Color Hypercharge.

You knew it was coming. Define Y^C and I_3^C appropriately...

The strong interaction strictly confines color. Thus any composite system observed in nature... hadrons, etc... is completely color neutral, colorless. $Y^C = I_3^C = 0$.

As a result it is impossible to create a free quark or a real propagating gluon.

Supersymmetry:

Quantum field theory has an inherent problem not associated with the pointlike nature of fundamental particles, or the filling of the vacuum with virtual particles. To understand it, let's play around with what's called Fock Space, or "Occupation Number Space." First consider operators which create and destroy bosons:

$$a^\dagger|0\rangle = |1\rangle,\ a|1\rangle = |0\rangle.$$

The vacuum is defined by $a|0\rangle = 0$.

Quantum field theory basically puts a harmonic oscillator at every point in space (arbitrary choice of basis) so let's investigate $H_B = (1/2)[a^\dagger a + aa^\dagger]$, where we impose the commutation relation $[a, a^\dagger] = \hbar\hat{1}$. So $H_B|0\rangle = (1/2)aa^\dagger|0\rangle = (1/2)(a^\dagger a + \hbar)|0\rangle = (\hbar/2)|0\rangle$. Therefore $E_{\text{B, vac}} = \hbar/2$.

Of course, as is well known, the harmonic oscillator eigenvalues are $E_n = \hbar(n + 1/2),\ n \geq 1$ in steps of 1. [We are setting $\omega = 1$.] Therefore it should come as no surprise that the vacuum (no particles) has energy $E_{\text{B, vac}} = \hbar/2$. This is the root of the problem. And it gets worse.

What about fermions? In this case,

$$H_F = (1/2)[\alpha^\dagger\alpha - \alpha\alpha^\dagger].$$

And we have an *anticommutation relation*, $\{\alpha, \alpha^\dagger\} = \hbar\hat{1}$.

Again, $\alpha|0\rangle = 0$.

Now $H_F|0\rangle = -(1/2)\alpha\alpha^\dagger|0\rangle = -(1/2)(\hbar - \alpha^\dagger\alpha)|0\rangle = -\hbar/2|0\rangle$.

So we still have the problem, $E_{\text{vac, F}} = -\hbar/2$.

The solution proposed is that the distinction between fermions and bosons is a *broken symmetry*. In other words, physics should not distinguish between bosons and fermions, but in our present universe it does.

This means that a supersymmetric oscillator should be like

$$H_s = a^\dagger a + \alpha^\dagger \alpha.$$

Then $H_s|0\rangle = 0$ and $E_{\text{vac, S}} = 0$. Problem solved, by doubling the number of known particles. Each boson has a fermion superpartner, and each fermion has a boson superpartner. With what mass? How big is the level splitting that occurs when the degeneracy is broken?? A natural energy scale is given by the Planck Mass, which is about 10^{19} GeV. There is a huge gap between that mass and the observed electroweak mass scale defined by the Higgs boson and $W^\pm$, Z^0 mass scale of $\simeq 100$ GeV.

Most folks who advocated supersymmetry expected the superpartners to appear where? "Even though accurate predictions of the the superpartner masses do not exist, there are three distinct arguments that make qualitative predictions of the masses. All three of them lead to the conclusion that a typical superpartner mass should be in the range of 100 GeV to 1000 GeV. In other words, they should be about 100–1000 times heavier than a proton." It is therefore not looking good!!

Two-Body Systems?

There are two "hydrogen-like" two body systems of interest in particle physics. One is positronium, which offers a nice test of QCD. The other is any meson made from very massive quarks; it can be treated non-relativistically in terms of a simple dynamical equation with a simple scalar potential $V(r)$.

Let's quickly review hydrogen:

$$[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{\alpha\hbar c}{r}]\psi(\mathbf{r}) = E\psi(\mathbf{r}).$$

Of course

$$E_n = \frac{-\alpha^2\mu c^2}{2n^2} \text{ with } \mu = \frac{m_p m_e}{(m_p + m_e)} \simeq m_e(0.51\ \text{MeV}/c^2).$$

Thus, as everybody knows,

$E_1 = -(1/137)^2(0.511\ \text{MeV})/2 \simeq -13.6\ \text{eV}.$

If we solve the Dirac equation instead, it accurately predicts "fine structure," breaking the degeneracy in ℓ via a spin-orbit term that naturally appears, and if we also take into account the proton spin, with a spin-spin interaction, breaking the j degeneracy, we get "hyperfine" structure. The splittings are small, typically 10^{-5} to 10^{-6} eV. This isn't all. If we quantize the electromagnetic field by including quantum fluctuations of the vacuum, we get additional level shifts, such as the famous "Lamb shift," also of order 10^{-6} eV. So H turns out to be a fine test-bed.

• Positronium

Now the reduced mass is about 0.255 MeV and the spin-spin term is much stronger since μ_e is about 650 times μ_p. We find a binding energy of about 7 eV and splittings of 10^{-3} eV. An important test of the approach is a correct prediction for

$$\Gamma(^1S_0 \to 2\gamma) = \frac{4\pi\alpha^2\hbar^3}{m_e^2 c}|\psi(0)|^2.$$

• Mesons composed of heavy quarks

One of the earliest examples was charmonium, c-$\bar{c}$. Great success was obtained in predicting the low-lying excitations of the system, using the obvious potential energy form

$$V(r) = -\frac{4\alpha_s\hbar c}{3r} + kr.$$

Typical values were $\alpha_s \simeq 0.2$, $k \simeq 1$ GeV/fm and m_c taken as an adjustable parameter (effective mass, to include medium effects). α_s was also taken as a weak function of r in some cases.

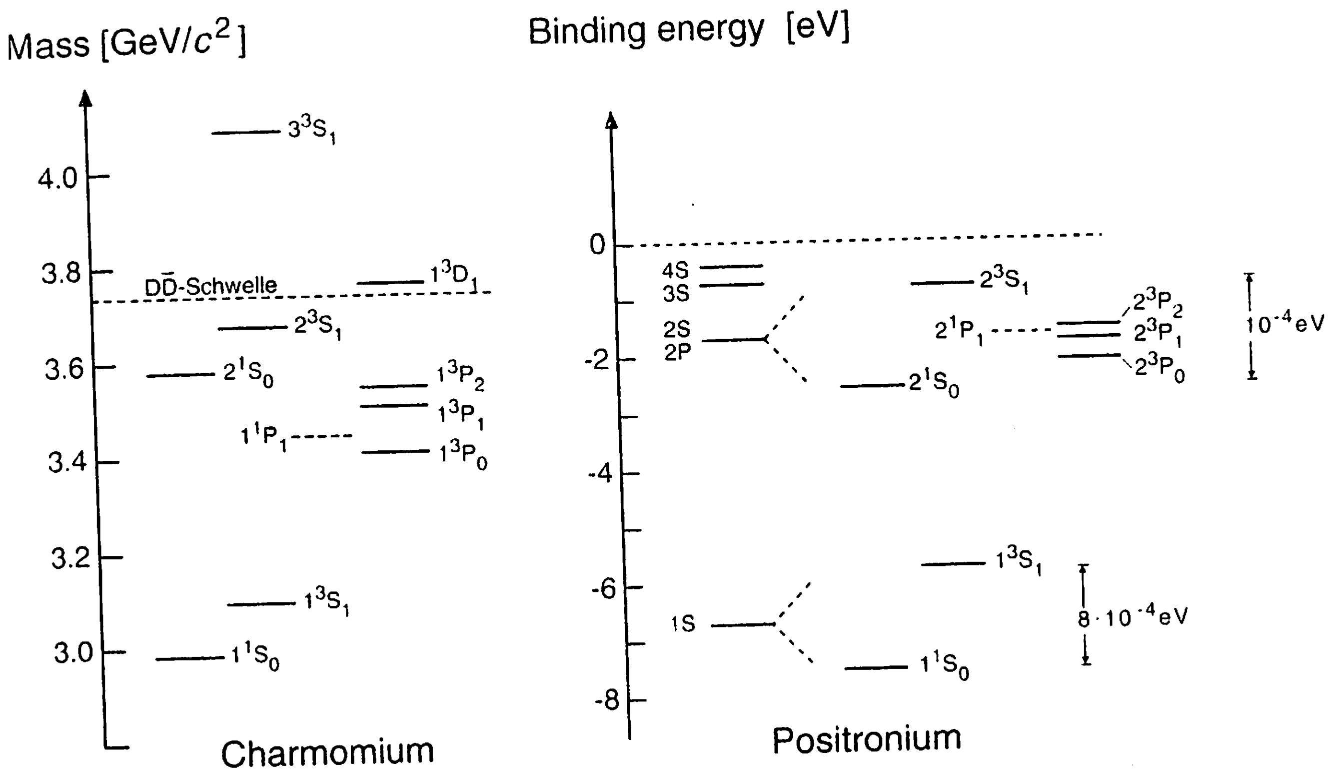

Fig. 13.6. Comparison of the energy levels of positronium and charmonium. The energy scales were chosen such that the 1S and 2S states of the two systems coincide horizontally. As a result of the differences in nomenclature for the first quantum number, the 2P states in positronium actually correspond to the 1P levels in charmonium. The splitting of the positronium states has been magnified. Dashed states have been calculated but not yet experimentally detected. Note that the $n=1$ and $n=2$ level patterns are very similar, while the 2S–3S separations are distinctly different. The dashed, horizontal line marks the threshold where positronium breaks up and charmonium decays into two D mesons (see Sect. 13.6).

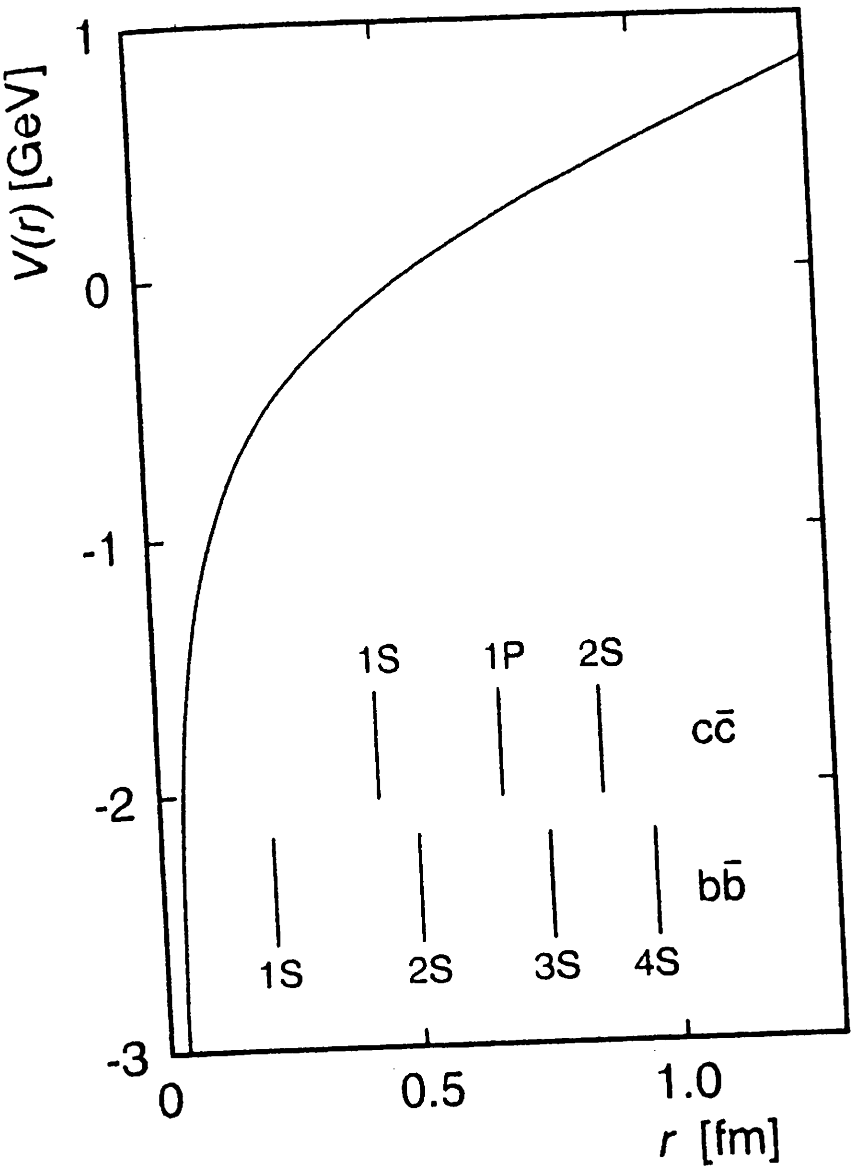

Fig. 13.8. Strong interaction potential versus the separation r of two quarks. This potential is roughly described by (13.6). The vertical lines mark the radii of the $c\bar{c}$ and $b\bar{b}$ states as calculated from such a potential (from [Go84]).

Constituent Quarks:

In the late 1960s it was common to try to understand hadrons using valence quarks only. The problem was immediately obvious. Consider the pi-mesons, made of various $\ell = 0$ combinations of u, d, $\bar{u}$ and $\bar{d}$. The mass of all the pions is about 140 MeV which would mean the average mass of u or d was around 70 MeV.

But consider the proton and neutron, made of $\ell = 0$ uud and udd states. Since the p has a mass of 938 MeV, you would think the average mass of u or d was around 310 MeV. Of course a spin-spin interaction could play a big role.

We now know thanks to decades of lattice gauge theory calculations that the masses of u, d, and s are about 2.3, 4.8 and 95 MeV, respectively.

But this just means that the valence quarks exist in a complex, fairly "rigid" field created by gluons and virtual quark-antiquark pairs, and we know from condensed matter physics that particles in such environments have an "effective mass" that is very different from their "bare mass." So the idea of the constituent quark models is to adopt effective masses, and include spin-spin terms where necessary.

If you write $M = m_q + m_{\bar{q}} + \Delta(\mathbf{S}_q, \mathbf{S}_{\bar{q}})$, this works surprisingly well for mesons, with the masses of u and d about 310 MeV and the mass of s about 483 MeV.

A similar expression for baryons,

$$M = \sum_i m_i + \Delta(\mathbf{S}_{q1}, \mathbf{S}_{q2}, \mathbf{S}_{q3}),$$

works well with $m_{u,d}$ about 363 MeV and m_s about 538 MeV.

Magnetic Moments?

We expect since the state is $\ell = 0$ that $\vec{\mu}_p = 2\vec{\mu}_u + \vec{\mu}_d$.

The spin state that results in spin-1/2 for three spin-1/2 particles you might remember from 373:

$$\psi_p(1/2, 1/2) = \Big[\sqrt{2/3}\chi_{uu}(1,1)\chi_d(1/2,-1/2)$$

$$-\sqrt{1/3}\chi_{uu}(0,0)\chi_d(1/2,1/2)\Big].$$

If we define $\mu_{u,d} = [Z_{u,d}e\hbar]/[2m_{u,d}]$, where $Z_u = 2/3$, $Z_d = -1/3$, then

$(\mu_u = -2\mu_d)$

$$\mu_p = (2/3)((2\mu_u - \mu_d) + (1/3)\mu_d) \simeq (3/2)\mu_u.$$

Now as is well known, $\mu_p = 2.79\mu_N = 2.79(e\hbar)/(2M_p)$.

Then $\mu_p \simeq (3/2)\mu_u = (e\hbar)/(2m_u)$.

This gives us a very interesting result, namely $m_u \simeq M_p/(2.79) \simeq 336$ MeV!

Of course the actual origin of the spin and magnetic moment of the proton and neutron is the object of extensive current research. It is remarkable that a "spin" of 1/2 emerges as the total angular momentum of a system dominated by particles with spin 1!!

TABLE 6.6 Values of the colour charges I_3^C, Y^C for the colour states of quarks and antiquarks

(a) Quarks			(b) Antiquarks		
	I_3^C	Y^C		I_3^C	Y^C
r	1/2	1/3	$\bar{r}$	−1/2	−1/3
g	−1/2	1/3	$\bar{g}$	1/2	−1/3
b	0	−2/3	$\bar{b}$	0	2/3

★ 6.3.2 Colour wavefunctions and the Pauli principle

In this section we give explicit expressions for the colour wavefunctions and operators, and show that for baryons the totally antisymmetric colour wavefunction (6.31) is not only allowed by colour confinement, but is required by it.

The three independent colour wavefunctions $\chi^C = r, g, b$ of a quark are conveniently represented by the 'colour spinors'

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{6.35}$$

in the same way that the two spin wavefunctions $\chi = \alpha, \beta$ were represented by the spinors

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

in section 4.2.1. Just as the spin wavefunctions are acted on by spin operators, the colour wavefunctions are acted on by 'colour operators'. The latter are represented by three-dimensional matrices in the same way that the spin operators are represented by the two-dimensional matrices (4.21). There are eight such independent colour operators

$$\hat{\mathrm{F}}_i = \tfrac{1}{2}\lambda_i \qquad (i = 1, 2, \ldots, 8) \tag{6.36a}$$

where the matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{6.36b}$$

can be regarded as the three-dimensional analogues of the two-dimensional Pauli matrices (4.21).* The colour wavefunctions (6.35) are eigenfunctions of the operators $\hat{F}_3$ and $\hat{F}_8$ with, for example,

$$\hat{F}_3 r = \frac{1}{2} r, \qquad \hat{F}_8 r = \frac{1}{2\sqrt{3}} r \tag{6.37}$$

for the r-state. The colour charges I_3^C and Y^C discussed in the previous section are the eigenvalues of the operators

$$\hat{I}_3^C \equiv \hat{F}_3, \qquad \hat{Y}^C \equiv \frac{2}{\sqrt{3}} \hat{F}_8, \tag{6.38}$$

where the factor $2/\sqrt{3}$ was introduced for historical reasons. From (6.37) and (6.38) it follows that $I_3^C = 1/2$ and $Y^C = 1/3$ for the r-state, and the corresponding values for the g and b states listed in Table 6.6(a) are obtained in a similar fashion. The remaining operators $\hat{F}_1$, $\hat{F}_2$, $\hat{F}_4$, $\hat{F}_5$, $\hat{F}_6$ and $\hat{F}_7$ mix the colour states (6.35), and one easily shows using (6.36) that, for example,

$$\hat{F}_1 r = \tfrac{1}{2} g, \qquad \hat{F}_1 g = \tfrac{1}{2} r, \qquad \hat{F}_1 b = 0. \tag{6.39}$$

The observables associated with the operators $\hat{F}_i$ are all believed to be exactly conserved in nature; i.e.

$$[\hat{F}_i, H] = 0 \qquad (i = 1, 2, \ldots, 8)$$

and for this reason they are all called colour charges. Colour confinement is the requirement that all eight colour charges vanish for any observed hadron h, implying

$$\hat{F}_i \chi_h^C = 0 \qquad (i = 1, 2, \ldots, 8) \tag{6.40}$$

for the corresponding colour wavefuncton χ_h^C. This is reminiscent of a spin singlet state, in which all th...

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{6.36b}$$

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and for this reason they are all called colour charges. Colour confinement is the requirement that all eight colour charges vanish for any observed hadron h, implying

$$\hat{F}_i \chi_h^C = 0 \qquad (i = 1, 2, \ldots, 8) \tag{6.40}$$

for the corresponding colour wavefuncton χ_h^C. This is reminiscent of a spin singlet state, in which all three spin components vanish, so that the spin wavefunction satisfies

$$\hat{S}_x \chi = 0, \qquad \hat{S}_y \chi = 0, \qquad \hat{S}_z \chi = 0, \tag{6.41}$$

It is for this reason that colour states which satisfy (6.40) are called colour singlets. Equation (6.40) implies, but is more restrictive than, the conditions (6.29) which were exploited extensively in the previous section.