Ultra-Relativistic Nuclear Collisons:

The goal of experiments has generally been to reach the region of the phase transition to a quark-gluon plasma, which should occur at about

$$T = 300 \text{ MeV/k} = 3.3 \times 10^{12} \text{ K}.$$

The corresponding energy density is about 10 GeV per cubic fermi, and the pressure is 0.5×10^{31} bar. So far experiments have been carried out at RHIC in the US and at the LHC in Europe. A new accelerator is now under construction in Germany, called FAIR. The reason for the intensive effort is that this is the only way anyone has thought of to probe the nuclear equation of state at extreme temperatures. [Nuclei themselves are of course close to $T = 0$.]

Almost all research of this kind began in Germany and most senior researchers in the field were educated in Germany. The theoretical description of the reaction events for obvious reasons depends on relativistic thermodynamics.

The usual theoretical approach to the nuclear equation of state is lattice gauge QCD. In any event, one begins with the QCD Lagrangian, translates it into a Hamiltonian, and constructs the usual thermodynamic partition function,

$$Z(T,V) = \mathrm{Tr}[\exp(-\widehat{H}/T)].$$

The energy density and pressure are then computed in the usual way, as

$$\varepsilon = [T^2/V](\partial Z/\partial T)_V,\ p = \frac{\partial \ln Z}{\partial V}|_T.$$

Early calculations (circa 1985) predicted the transition temperature to be about 175 to 200 MeV, corresponding to an energy density of about 2.5 GeV per cubic fermi. There may be a critical point around 155 MeV, but perhaps at higher density than RHIC and the LHC can directly reach. RHIC and the LHC have easily reached temperatures of 300 to 400 MeV, but more density is the goal in future work.

Before experiments began, there were several (unfortunately very indirect) experimental indicators suggested for a quark-gluon plasma, and in early experiments up to 2000, mainly done at RHIC, every single one of these indicators was unambiguously observed. Alas, and alack, when energies were dropped BELOW the reliably predicted position of the phase transition, *the same "indicators" were still observed!* This has left the field in turmoil, because it is no longer clear how to identify the quark-gluon phase in experiments. There is no question that the proper degree of quark deconfinement has been achieved, even in early studies, but at present no one has a good suggestion as to how to verify this experimentally!

A new proton-antiproton collider, FAIR, is currently under construction in Germany, which may be able to probe regions of the equation of state that are inaccessible to heavy-ion collision experiments. In any event, comparisons of p + p, p + nucleus, and nucleus + nucleus collisions have already been invaluable in understanding what's been found over the last 30 years.

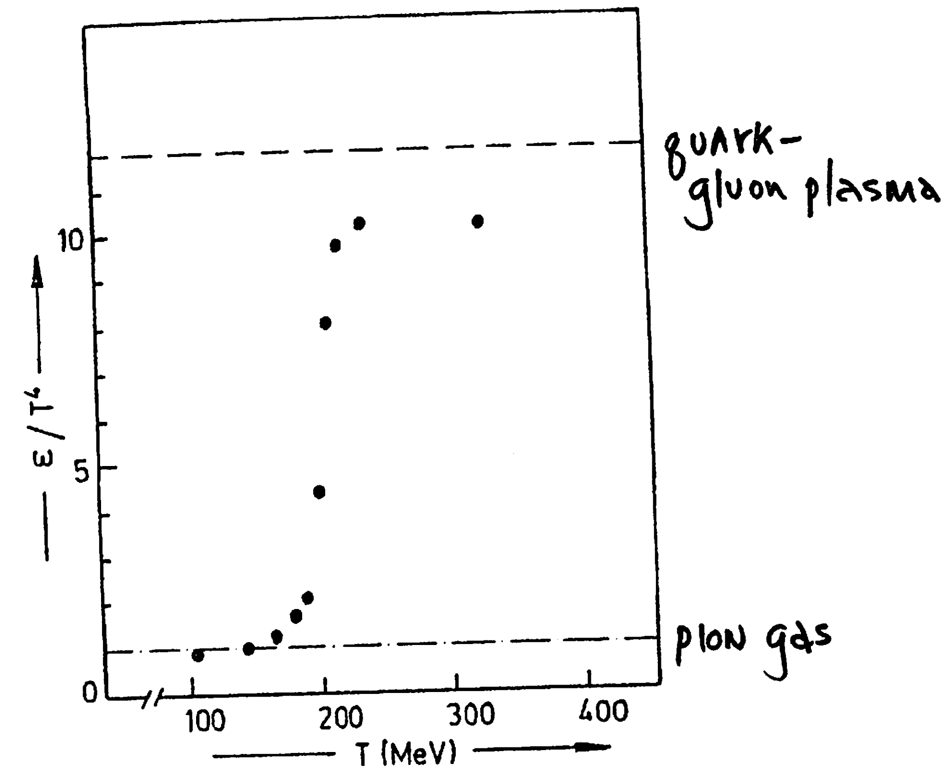


Figure 15.10. The energy density (ε/T^4) of strongly interacting matter as a function of the temperature. We also compare the horizontal lines for the ideal quark–gluon plasma (dashed line) and for the ideal pion gas (dot-dashed line) (taken from Celik *et al* 1985).

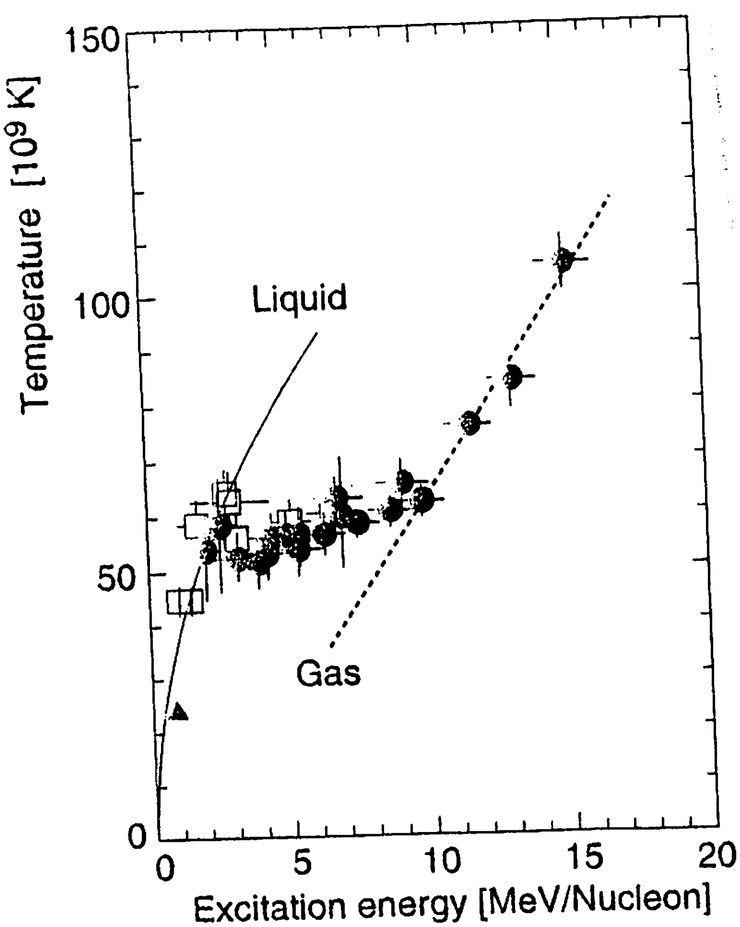


Fig. 19.6. Temperature of the fragments in a peripheral collision of two ^{197}Au nuclei as a function of the excitation energy per nucleon (from [Po95]). The behaviour of the temperature can be understood as a phase transition in nuclear matter.

UNSTABLE NUCLEI:

It's a feature of quantum physics that transition probabilities are time-independent, and in such a case

$$N(t) = N(0)\exp(-t/\tau) = N(0)\left[\frac{1}{2}\right]^{t/T_{1/2}}.$$

Various processes are possible by which unstable nuclei can transform to nuclei of greater stability:

(1) Alpha decay— a ^{4}He cluster penetrates the Coulomb barrier.

(2) Beta-minus decay— a neutron converts into a proton, electron and anti-electron neutrino.

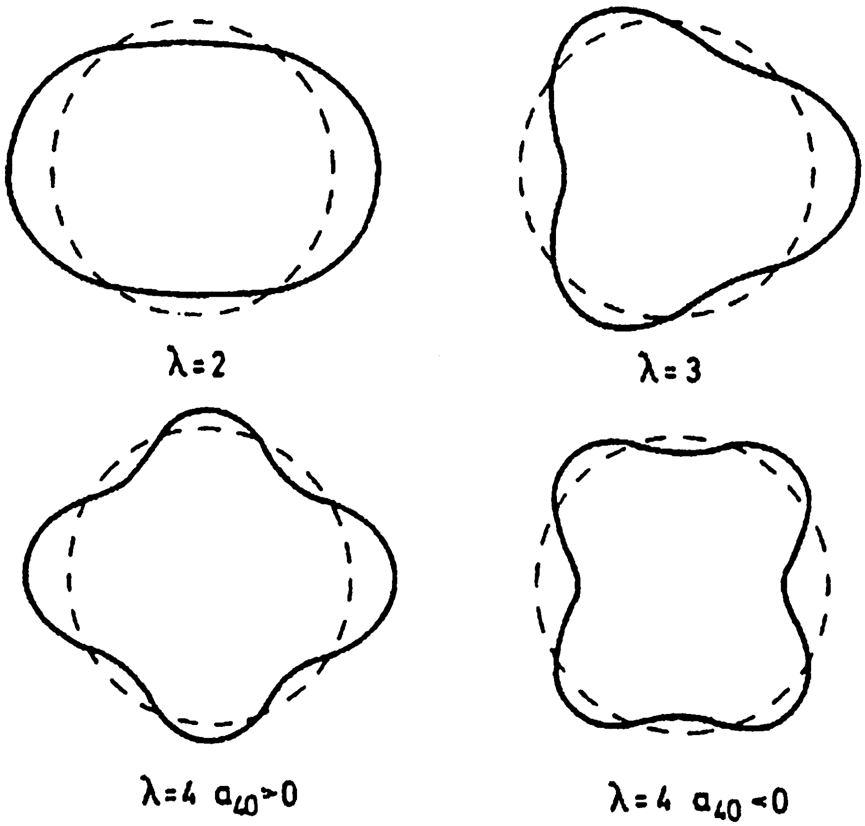
(3) Beta-plus decay— a proton converts into a neutron, positron and electron neutrino.

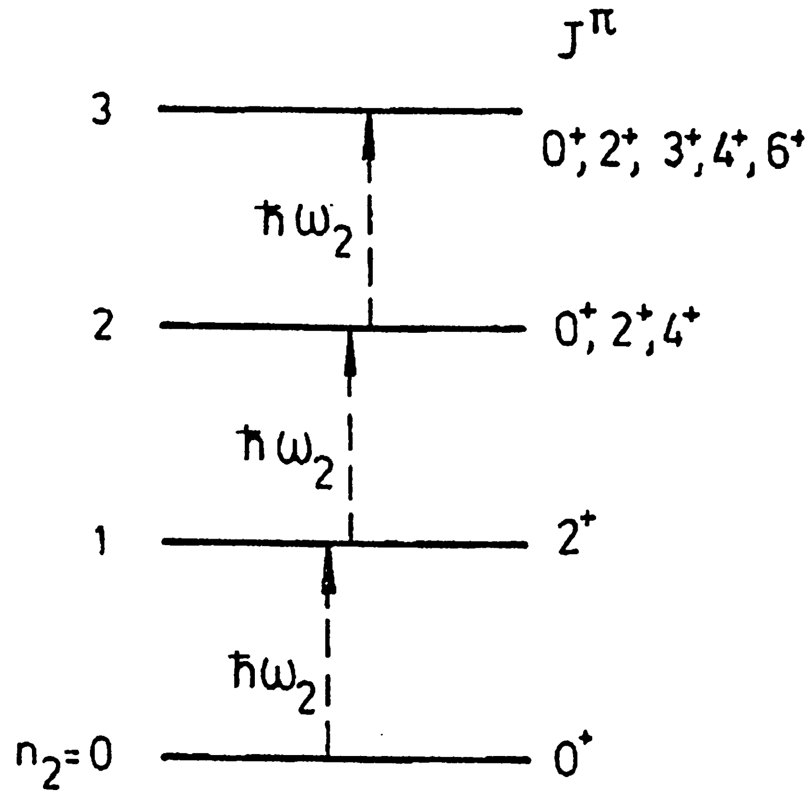
(4) Electron capture— a K electron is absorbed by a proton, yielding a neutron and an electron neutrino.

Processes (3) and (4) are always competitive except in very light nuclei.

(5) Spontaneous fission— in very heavy nuclei; because of the huge Coulomb barrier this process is not always probable.

(6) De-excitation of excited nuclear states, after decays (1) through (5):


λ=2
λ=3
λ=4 $a_{40}>0$
λ=4 $a_{40}<0$


J^{π}
3
$0^+, 2^+, 3^+, 4^+, 6^+$
$\hbar\omega_2$
2
$0^+, 2^+, 4^+$
$\hbar\omega_2$
1
2^+
$\hbar\omega_2$
$n_2=0$
0^+

Vibrational Levels in Nuclei

In mid-shell, spontaneous symmetry breaking produces permanent nuclear deformations. Otherwise, even-even nuclei have spherical symmetry, and their collective modes are vibrational.

Let's write

$$R(\theta, \phi) = R_0 \Big[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta\phi) \Big].$$

• $\lambda = 0$ would be a density oscillation ("breathing") but nuclei are essentially incompressible.

• $\lambda = 1$ would be a translation (equivalent to a Goldstone Boson), but one does see combinations of translations at high excitation (Giant Dipole Resonance, etc.).

So we mainly encounter $\lambda \geq 2$.

If we define

$$H_{\text{vib}} = \sum_{\lambda\mu} (B_\lambda/2) |\dot{\alpha}_{\lambda\mu}|^2 + \sum_{\lambda\mu} (C_\lambda/2) |\alpha_{\lambda\mu}|^2,$$

we can define new operators in terms of $\alpha_{\lambda\mu}$ and $\dot{\alpha}^*_{\lambda\mu}$ that satisfy $[b_{\lambda'\mu'}, b^\dagger_{\lambda\mu}] = \delta_{\lambda\lambda'}\delta_{\mu\mu'}$.

Then a transformation results in

$$H_{\text{vib}} = \sum_{\lambda} \hbar\omega_{\lambda} \sum_{\mu} \left(b^{\dagger}_{\lambda\mu} b_{\lambda\mu} + \frac{1}{2} \right).$$

Here $\omega_{\lambda} = \sqrt{\frac{C_{\lambda}}{B_{\lambda}}}$.

Each vibrational phonon is an angular momentum eigenstate, $|\lambda\mu\rangle$. Coupling is via the usual Clebsch-Gordon coefficients.

Vibrational states were mainly studied in the early days of nuclear physics by electromagnetic transitions.

If we define $M(E\lambda, \mu) = (Ze/A) \int d^3r r^{\ell} Y_{\ell m}(\widehat{\mathbf{r}})\rho(r)$, we can expand

$$\rho(\mathbf{r}) \simeq \rho_0(r) - R_0(\partial\rho_0/\partial r) \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^{*}_{\lambda\mu}(\widehat{\mathbf{r}}).$$

Then it is possible to obtain $M(E\lambda, \mu)$ in terms of the b operators.

It is customary to define a reduced transition probability $B(E\lambda, \lambda \to 0^+)$ which is proportional to $|M(E\lambda, \mu)|^2$, and in the earliest days of nuclear physics it became obvious that for true vibrational states $B(E2)$

is many, many times the value that a single-nucleon transition would predict, making clear that the nuclear states involved were indeed vibrations involving many nucleons.

Giant Dipole Resonance:

Experimentally $E_R(1^-)$ is at 79 MeV$/A^{1/3}$. It can be understood as an oscillation of the proton and neutron densities relative to one another. Define $\rho_0 = \rho_p + \rho_n$ and $\delta\rho_p = \rho_p - (\rho_0/2)$. Then

$$\nabla^2 \delta\rho_p - (1/v^2)\frac{\partial^2 \delta\rho_p}{\partial t^2} = 0.$$

Defining $k = \omega/v$, we get

$$\nabla^2 \delta\rho_p + k^2 \delta\rho_p = 0.$$

We are interested in $\lambda = 1$ so the solution we want is

$$\delta\rho_p \propto j_1(kr) Y_{1\mu}(\widehat{\mathbf{r}}).$$

To get a standing wave impose

$$(d/dr) j_1(kr)|_{r=R} = 0.$$

The result is $kR = 2.08$.

The semi-empirical mass formula suggests the speed of sound is around 73 fm/10^{-21} sec. and if we use $R = 1.2A^{1/3}$ fm, the very simple prediction is

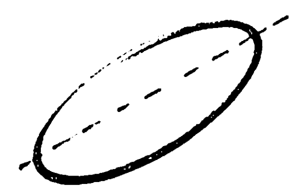
$$82A^{-1/3} \text{ MeV}.$$

A number of other giant resonances are observable, such as the quadrupole and octopole versions. A particular target of interest is the giant monopole resonance, which would allow some insight into nuclear compressibility.

Nuclear Rotations:

When shells are about half-filled, spherical symmetry is badly broken and nuclei display permanent deformations. The nucleus thus develops a symmetry axis. Suppose we could locate this axis with an angle φ. Then $L_\varphi = -i\hbar\partial/\partial\varphi$, which leads to the usual uncertainty relation

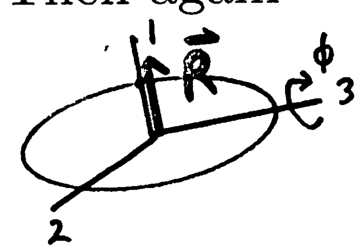
$$\Delta\varphi\Delta L_\varphi \geq \hbar.$$



But the rotational spectra we observe give some degree of information about a possible range of orientations, so we expect a broad band of corresponding L-states.

Call the symmetry axis the body-fixed 3-axis. If ϕ rotates the system about the symmetry axis, then $\Delta\phi \to \infty$. Define $R_3 = -i\hbar(\partial/\partial\phi)$. Then again

$$\Delta R_3 \Delta\phi \geq \hbar.$$



This means $\Delta R_3 \to 0$.

Let's take **R** along axis 1, perpendicular to 3. We construct a rigid-rotor Hamiltonian,

$$H_{\text{rot}} = \frac{\widehat{R}^2}{2\mathcal{I}}, \text{ with } \mathcal{I} \text{ the rotational inertia.}$$

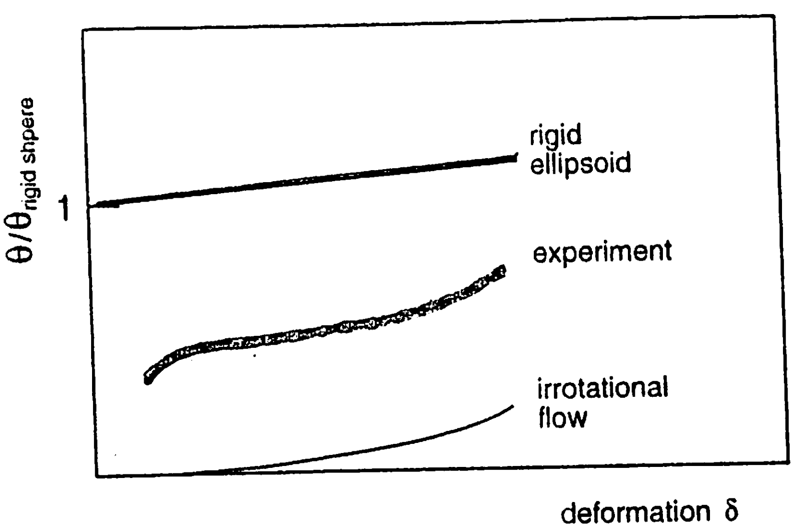


Fig. 18.13. Moments of inertia of deformed nuclei compared with a rigid sphere as a function of the deformation parameter δ. The extreme cases of a rigid ellipsoid and an irrotational liquid are given for comparison.

Then, in terms of the usual $|JM\rangle$ states,

$$\widehat{R}^2|JM\rangle = J(J+1)\hbar^2|JM\rangle.$$

Remember the parity of an angular momentum eigenstate is $(-1)^J$. Because the deformation is invariant about 1-2 plane reflection, only positive parity states are allowed, so J must be even.

Therefore $E_J = (\hbar^2 J(J+1))/(2\mathcal{I}),\ \ J = 0, 2, 4,$ etc. If the rotational inertia does not depend on J (actually it does, as J gets large), then E_J can be expressed very simply in terms of E_2.

We could model the rotational inertia of the system as that of a solid ellipsoid, a rigid rotor. Or we could say that since the deformed nuclei occur in the middle of shells, there is an irrotational spherical core of paired nucleons, and a deformed atmosphere of unpaired nucleons surrounds it. In reality, the measured rotational inertias lie about halfway between predicted values for rigid and irrotational systems.

When we examine single-nucleon states outside a deformed core (odd-A systems) we get insanely complex spectra which have mainly been explored, like rotational nuclei themselves, not by nuclear physicists, but rather by nuclear chemists. The deformation breaks a degeneracy and leads to a new quantum number K. [See text, pp. 551 - 558, 18.3 and 18.4.)

When we try to describe nuclear deformation in quantum physics we encounter another serious conceptual problem, namely that the symmetry axis cannot possibly be a fixed axis in space. This leads to the introduction of the infamous rotation matrix $D^{\lambda}_{\mu\mu'}(\Omega)$, which depends on the Euler angles.

This rotation matrix lets us relate the intrinsic, body-fixed deformation parameters to the observed "lab"-system parameters.

For an axially symmetric deformation we need only one parameter, usually called β_{λ}. For quadrupole deformations, two parameters are needed, $\alpha_{20} = \beta \cos\gamma$ and $\alpha_{22} = (2)^{-1/2} \beta \sin\gamma$. These parameters, and Y_{20} and $Y_{2,\pm 2}$ allow one to write $R(\theta, \phi)$ in terms of θ, ϕ, β and γ.

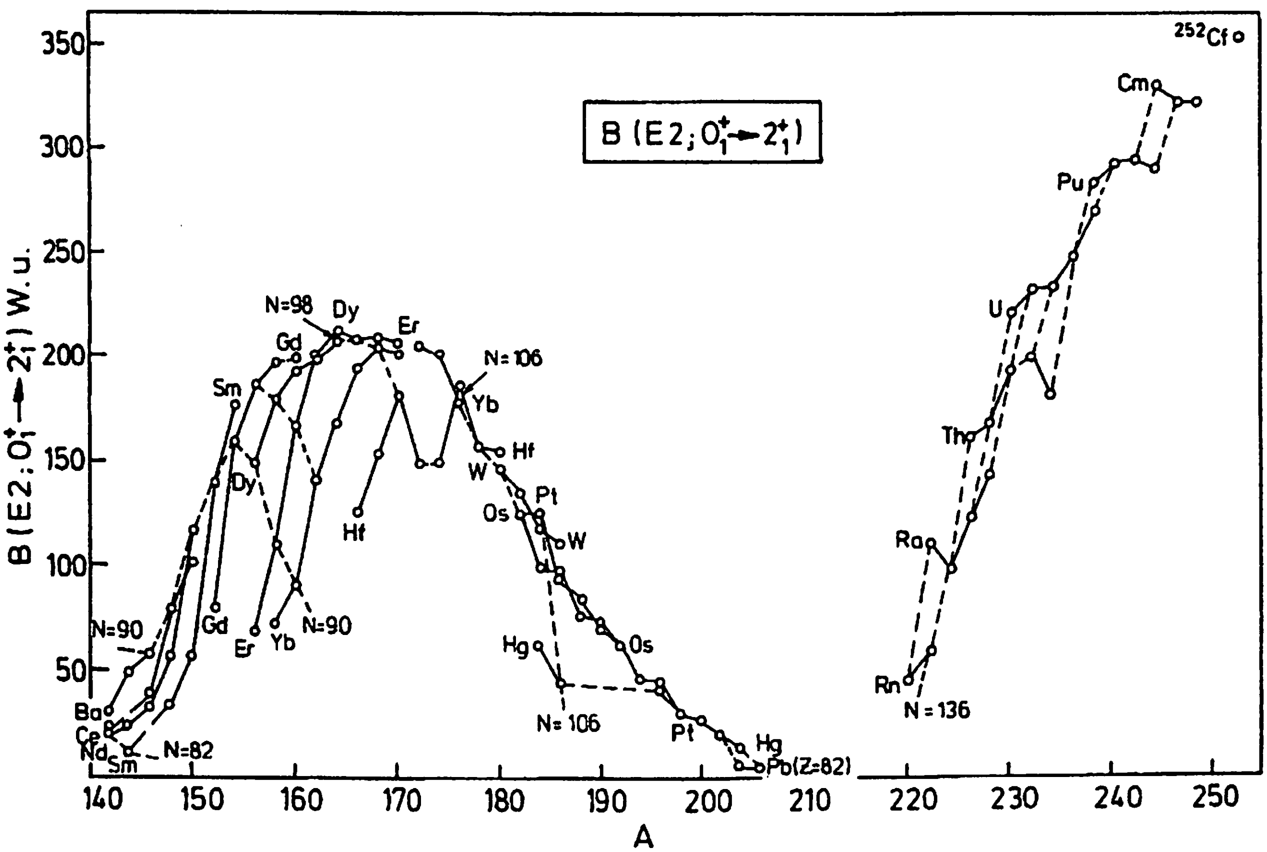


Figure 12.2. Systematics of the $B(E2; 0_1^+ \rightarrow 2_1^+)$ values for the even–even nuclei with $N \geq 82$, $Z \leq 98$. The $B(E2)$ values are expressed in Weisskopf units (WU) (taken from Wood 1992).