Motion along a line!

Average velocity:

$$\bar{v}_x = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

This is not a very useful idea, what we need is the *instantaneous* velocity $v_x(t)$, which is the slope of the function $x(t)$ at each instant.

$$v_x(t) = \frac{dx}{dt}.$$

Average acceleration:

$$\bar{a}_x = \frac{v_x(t_2) - v_x(t_1)}{t_2 - t_1}.$$

Again, what we actually need is the *instantaneous* acceleration $a_x(t)$, which is the slope of the function $v_x(t)$ at each instant.

$$a_x(t) = \frac{dv_x}{dt}.$$

Constant Acceleration:

$$\bar{a}_x = a_x = \frac{v_x(t) - v_x(0)}{t - 0},$$

so

$$v_x(t) = v_x(0) + a_x t.$$

Since $dx/dt = v_x$ we instantly see that

$$x(t) = x(0) + v_x(0)t + \frac{1}{2}a_x t^2.$$

If we solve the $v_x(t)$ equation for t and insert it into the $x(t)$ equation, we find after some algebra that

$$v_x^2 - v_x(0)^2 = 2a_x(x - x(0))$$

which is a very useful result when you need v_x as a function of x instead of t.

FALLING:

A classic example of motion under constant acceleration, discovered by Galileo, is the motion of an object unsupported in the earth's atmosphere. Such a body has a constant acceleration $g = 10$ m/s^2 toward the center of the earth.

If we call the vertical axis y then

$$y(t) = y(0) + v_y(0)t - \frac{1}{2}gt^2.$$

$$v_y(t) = v_y(0) - gt.$$

and we can also write

$$v_y^2 - v_y(0)^2 = -2g(y - y(0)).$$

COLLISIONS:

Consider two independent objects described by $x_1(t)$ and $x_2(t)$ or $y_1(t)$ and $y_2(t)$. The objects will collide at a time when $x_1 = x_2$ or $y_1 = y_2$.