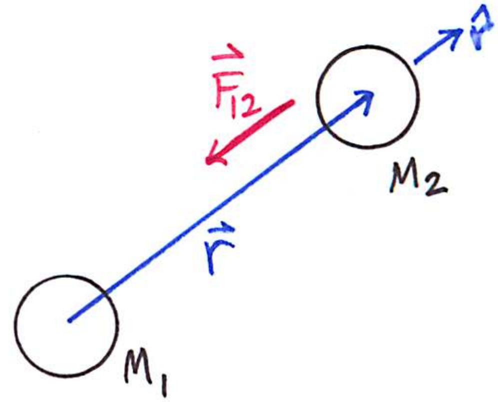
GRAVITY!



$$\mathbf{F}_{12} = -\widehat{\mathbf{r}}\frac{GM_1M_2}{r^2}.$$

The universal constant G has a value of about 6.7 $\times 10^{-11}$ N-m^2/kg^2.

- ***Circular Orbit of Earth:***

$$v_c = R_e\sqrt{\frac{g}{r}} = \sqrt{\frac{GM_e}{r}}.$$

- ***Kepler's 3rd Rule:***

$$T^2 \propto r^3.$$

- ***Elliptical Orbit:***

$$r_p = a(1-e),\ r_a = a(1+e),\ b = a\sqrt{1-e^2},$$

Also $e = \sqrt{1-(b/a)^2}$ and $v_p r_p = v_a r_a$.

The only orbits actually found in nature are perturbed elliptical and perturbed hyperbolic orbits.

- ***Gravitational Potential Energy:***

$$U(r) = -\frac{GMm}{r} \text{ or for earth, } U(r) = -mgR_e^2/r.$$

- ***Escape Speed:***

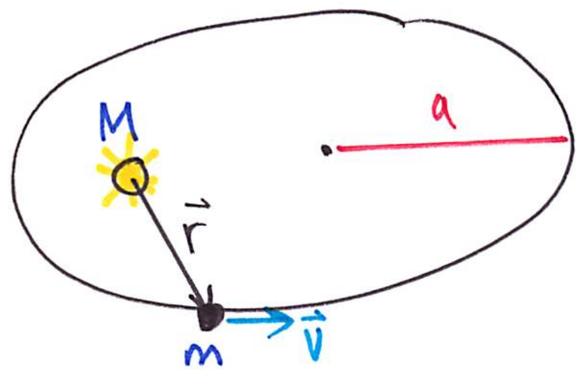
For earth, $v_e = \sqrt{2gR_e}$, in general $v_e = \sqrt{2GM/r}$.

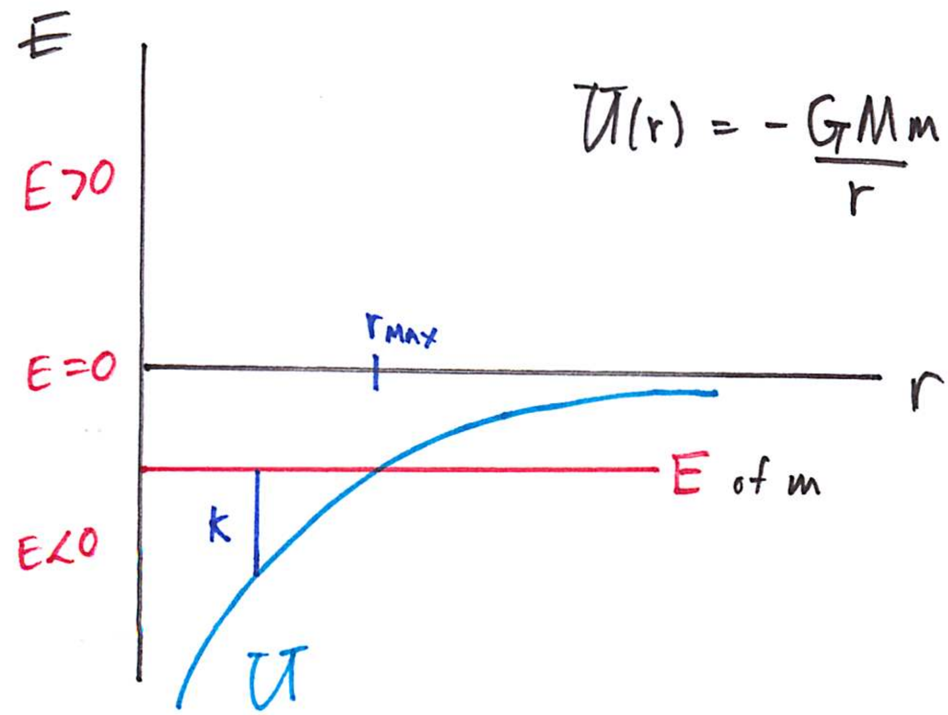
- ***Circular Orbit Energy Relations:***

$$K = -U/2,\ E = -K,\ E = U/2.$$

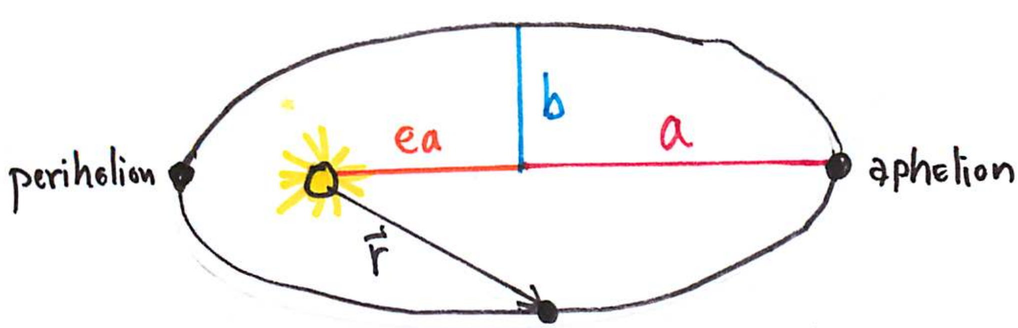
- ***Elliptical Orbit:***

$$E = -\frac{GMm}{2a} \text{ and } E = \frac{1}{2}mv^2 - \frac{GMm}{r}.$$





If the energy $E = K + U$ of mass m relative to mass M is less than zero, then m is "bound" to M. That is, if we throw m directly away from M at some initial speed, it will come to rest at some point $r_{\rm max}$ and then fall back. However, if $E = 0$ there is no finite r for which the kinetic energy of m goes to zero... it "escapes" M. For any $E > 0$ m has a large amount of kinetic energy even at infinity.



For an elliptical orbit, a, the semimajor axis, is half the longest distance across the ellipse, and b, the semiminor axis, is half the shortest distance across the ellipse. The eccentricity e of the ellipse measures the difference between a and b, and is zero when $a = b$, a circular orbit. The distance of the star from the center of the ellipse is ae, and two other important points are the perihelion distance $r_p = a(1 - e)$, and the aphelion distance, $r_a = a(1 + e)$, the points of closest approach and furthest distance of the planet from the star. Note $r_p + r_a = 2a$. Kepler also noticed that for the perihelion and aphelion speeds, $v_p r_p = v_a r_a$, in other words the planet moves slowest at the aphelion and fastest at the perihelion, not surprising since these are the points of greatest and least U, respectively. As we said in class, the eccentricity can be defined by $e^2 = 1 - (b/a)^2$.