GRAVITY!

\[ \mathbf{F}_{12} = -\hat{r} \frac{GM_1 M_2}{r^2}. \]

The universal constant \( G \) has a value of about 6.7 \( \times 10^{-11} \) N-m\(^2\)/kg\(^2\).

- **Circular Orbit of Earth:**

\[ v_c = R_e \sqrt{\frac{g}{r}} = \sqrt{\frac{G M_e}{r}}. \]

- **Kepler’s 3rd Rule:**

\[ T^2 \propto r^3. \]

- **Elliptical Orbit:**

\[ r_p = a(1 - e), \quad r_a = a(1 + e), \quad b = a \sqrt{1 - e^2}, \]

Also \( e = \sqrt{1 - (b/a)^2} \) and \( v_p r_p = v_a r_a \).

The only orbits actually found in nature are perturbed elliptical and perturbed hyperbolic orbits.

- **Gravitational Potential Energy:**

\[ U(r) = -\frac{G M m}{r} \quad \text{or for earth, } U(r) = -mg R_e^2 / r. \]
• **Escape Speed:**
For earth, \( v_e = \sqrt{2gR_e} \), in general \( v_e = \sqrt{2GM/r} \).

• **Circular Orbit Energy Relations:**

\[
K = -U/2, \quad E = -K, \quad E = U/2.
\]

• **Elliptical Orbit:**

\[
E = -\frac{GMm}{2a} \quad \text{and} \quad E = \frac{1}{2}mv^2 - \frac{GMm}{r}.
\]
If the energy $E = K + U$ of mass $m$ relative to mass $M$ is less than zero, then $m$ is "bound" to $M$. That is, if we throw $m$ directly away from $M$ at some initial speed, it will come to rest at some point $r_{\text{max}}$ and then fall back. However, if $E = 0$ there is no finite $r$ for which the kinetic energy of $m$ goes to zero... it "escapes" $M$. For any $E > 0$ m has a large amount of kinetic energy even at infinity.

For an elliptical orbit, $a$, the semimajor axis, is half the longest distance across the ellipse, and $b$, the semiminor axis, is half the shortest distance across the ellipse. The eccentricity $e$ of the ellipse measures the difference between $a$ and $b$, and is zero when $a = b$, a circular orbit. The distance of the star from the center of the ellipse is $ac$, and two other important points are the perihelion distance $r_p = a(1 - e)$, and the aphelion distance, $r_a = a(1 + e)$, the points of closest approach and furthest distance of the planet from the star. Note $r_p + r_a = 2a$. Kepler also noticed that for the perihelion and aphelion speeds, $v_p r_p = v_a r_a$, in other words the planet moves slowest at the aphelion and fastest at the perihelion, not surprising since these are the points of greatest and least $U$, respectively. As we said in class, the eccentricity can be defined by $e^2 = 1 - (b/a)^2$. 