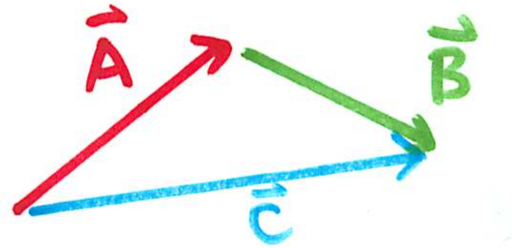
VECTORS!

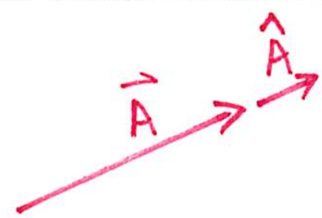
A vector is a quantity with both magnitude and direction. Vectors do not obey the rules of algebra; they add and subtract according to a parallelogram rule:



$\vec{A} + \vec{B} = \vec{C}$

A vector can be separated into a magnitude and a unit vector which supplies direction information.

$$\mathbf{A} = [10\ \mathrm{m}]\widehat{\mathbf{A}}.$$



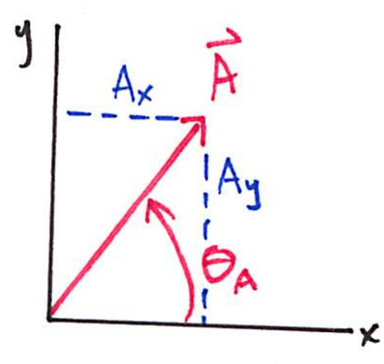
The vectors we will normally encounter in the course are position $\mathbf{r}$, velocity $\mathbf{v}$, acceleration $\mathbf{a}$ and force $\mathbf{F}$.

Components:

In the usual cartesian coordinate system we can define

$$A_y = A \sin\theta_A,\ A_x = A\cos\theta_A,\ A = \sqrt{A_x^2 + A_y^2},$$

$$\text{and}\ \tan\theta_A = \frac{A_y}{A_x}.$$



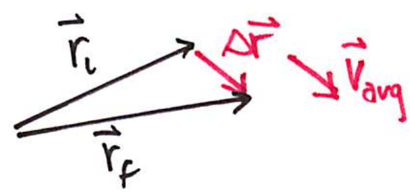
Cartesian Unit Vectors:

Unit vectors $\widehat{\mathbf{i}}$, $\widehat{\mathbf{j}}$ and $\widehat{\mathbf{k}}$ point along the x, y and z axes, respectively.

A vector in general can be written as

$$\mathbf{A} = \widehat{\mathbf{i}}A_x + \widehat{\mathbf{j}}A_y + \widehat{\mathbf{k}}A_z.$$

The use of unit vectors makes it trivial to add or subtract vectors.



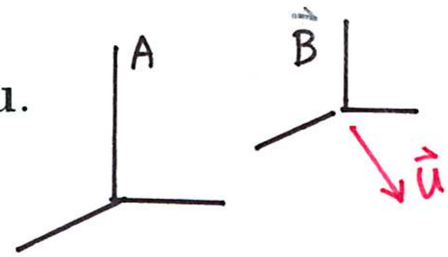
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \text{ and } \mathbf{a} = \frac{d\mathbf{v}}{dt}.$$

Note that $\mathbf{v}_{\text{avg}}$ is always in the direction of $\Delta\mathbf{r}$ and $\mathbf{a}_{\text{avg}}$ is always in the direction of $\Delta\mathbf{v}$.

Relative Velocity:

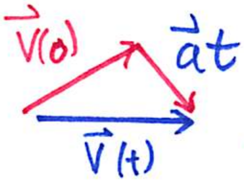
If the velocity of an object seen from frame of reference A is $\mathbf{v}_A$, and another frame of reference, B, is moving at velocity $\mathbf{u}$ with respect to A, the velocity of the same object seen from frame of reference B is:

$$\mathbf{v}_B = \mathbf{v}_A - \mathbf{u}.$$

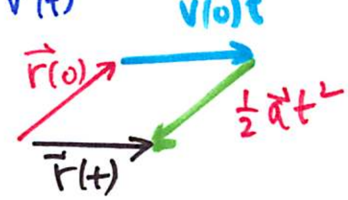


Constant Acceleration, in General:

$$\mathbf{v}(t) = \mathbf{v}(0) + \mathbf{a}t,$$



$$\mathbf{r}(t) = \mathbf{r}(0) + \mathbf{v}(0)t + \frac{1}{2}\mathbf{a}t^2.$$



In component form, we get the same equations for $x(t)$ and $y(t)$ that we used in Ch. 2.

Projectiles:

We use the above equations with $\mathbf{a} = \mathbf{g}$, where $g_x = 0$ and $g_y = -g$.

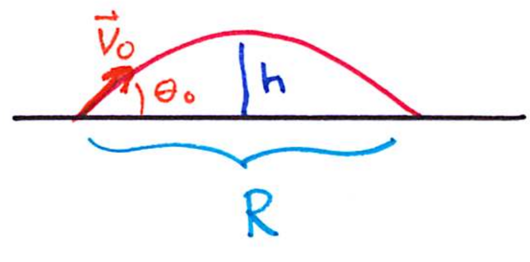
Thus for instance $x(t) = x(0) + v_x(0)t$ and $y(t) = y(0) + v_y(0)t - (1/2)gt^2$.

Using these equations it is very easy to show, for example, that the maximum height reached by a projectile is

$$h = \frac{(v_0 \sin\theta_0)^2}{2g}$$

and the maximum horizontal distance attained is

$$R = \frac{v_0^2 \sin(2\theta_0)}{g}.$$



Circular Motion: For motion in a circle of radius r at constant speed v, by definition of acceleration, we find

$$a = \frac{v^2}{r}.$$

Note that while the magnitude a of the acceleration $\mathbf{a}$ is constant, the directions of both $\mathbf{a}$ and $\mathbf{v}$ are constantly changing.