SIMPLE HARMONIC OSCILLATOR:

We say we have a simple harmonic oscillator if Newton's 2nd Law or 2nd Law for Torques looks as follows:

$$\text{acceleration parameter} =$$

$$-\text{constant} \times \text{displacement parameter}$$

For example: $a_x = -\omega^2 x$, or $\alpha = -\omega^2 \theta$.

The constant ω is called the *angular frequency* of the oscillator.

The easiest parameters to measure for periodic motion are the frequency in cycles per second (Hz), f, and the period in seconds per cycle, T. Obviously $T = (1/f)$.

- Mass on a spring on a frictionless horizontal surface:

$$m\frac{d^2x}{dt^2} = -kx.$$

We instantly see $\omega = \sqrt{k/m}$.

The solution to this differential equation, if the mass is pulled out to $x = A$ beyond equilibrium and released, would be

$$x(t) = A\cos[\omega t].$$

Note then that $v_x = -A\omega\sin[\omega t] = -v_{\max}\sin[\omega t]$ and that $a_x = -A\omega^2\cos[\omega t] = -a_{\max}\cos[\omega t] = -\omega^2 x$, of course.

Since the function is periodic, $\omega T = 2\pi$ so that $T = (2\pi)/\omega$ and $f = (\omega)/(2\pi)$.

Thus for a mass on a spring, $T = 2\pi\sqrt{m/k}$.

Oscillator Phase:

If we write $x(t) = A\cos[\omega t + \delta]$ we can start the oscillator at an arbitrary point. $x(0) = A\cos[\delta]$ and $v_x(0) = -A\omega\sin[\delta]$. Also it is easy to see that $\tan\delta = -v_x(0)/(\omega x(0))$.

Simple Pendulum: $m\ell^2\alpha \doteq -mg\ell\theta$, assuming $\sin\theta \approx \theta$.

Then clearly $T = 2\pi\sqrt{\ell/g}$.

Physical Pendulum: $I_p\alpha = -Mg\theta r_{\text{cm}}$, assuming $\sin\theta \approx \theta$.

Clearly then $T = 2\pi\sqrt{I_p/(Mgr_{\text{cm}})}$.

Energy Conservation: $E = K + U$.

For example for a spring, $v_x(x) = \pm\omega\sqrt{A^2 - x^2}$.

DRIVEN OSCILLATIONS AND RESONANCE:

Suppose

$$m\frac{d^2x}{dt^2} = -kx + F_0 \cos[\omega_0 t].$$

An obvious solution is $x(t) = A\cos[\omega_0 t]$. Plugging it in gives

$$A(\omega_0) = \frac{F_0/m}{\omega_0^2 - \omega^2},$$

where ω is the natural frequency $\omega = \sqrt{k/m}$. Notice that as ω_0 approaches the natural frequency of the system, the amplitude diverges! Resonance is a common property of almost all physical systems.

DAMPING:

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}.$$

The solution looks like

$$x(t) = A\exp(-bt/(2m))\cos[\omega t + \phi].$$

The Q for an oscillator is defined as

$$Q = \frac{2\pi E}{|\Delta E|}.$$

Overdamping: Q < 1/2. Critical Damping: Q = 1/2. Underdamping: Q > 1/2. Oscillators used in physics research can have Q of many, many thousands.

Driven Damped Oscillator:

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \cos[\omega_0 t].$$

Solution $x(t) = A(\omega)\cos[\omega_0 t + \phi]$.

$$A(\omega) = \frac{(F_0/m)}{\sqrt{(\omega_0^2 - \omega^2)^2 + (b\omega_0/m)^2}}.$$

To summarize: Suppose η is some kind of displacement parameter. Could be a distance, could be an angle, etc. If the 2nd Law or 2nd Law for Torques for the system looks like

$$\frac{d^2\eta}{dt^2} = -\omega^2\eta,$$

where ω is some constant, THEN THE SYSTEM IS A SIMPLE HARMONIC OSCILLATOR, and it oscillates with a period of

$$T = \frac{2\pi}{\omega}.$$

The solution to the 2nd Law would look, in the simplest case, like

$$\eta(t) = \eta_{\text{max}} \cos[\omega t].$$

Similarly, suppose the expression for the total energy of an oscillator has the form

$$E = B\left[\frac{d\eta}{dt}\right]^2 + C\eta^2,$$

where B and C are positive constants. THEN THE SYSTEM IS A SIMPLE HARMONIC OSCILLATOR with $\omega = \sqrt{C/B}$ and a period of

$$T = \frac{2\pi}{\omega}.$$