Key Concepts, Ch. 22

- **Coulomb’s Law** for the force between two charged objects. (+ or – makes a big difference!)

- **Systems of Charge**— applying Coulomb’s law when you have three or more charges.

- **Conservation of Charge**— charge is a scalar quantity which adds algebraically.

- **Conductors versus Insulators**— is some charge free or is all charge anchored?

- **Ways to charge objects**... Rubbing, dumping, induction, grounding, etc.

In almost all cases, all movable or transportable charge consists of fundamental point particles called electrons.
Gravity:

$$F = -G \frac{M_1 M_2}{r^2} \hat{r}.$$  

Coulomb Force:

$$F = +k \frac{Q_1 Q_2}{r^2} \hat{r}.$$  

The standard unit of mass is the kilogram. The standard unit of charge is the Coulomb.

In standard units, $G = 6.7 \times 10^{-11}$ N-m$^2$/kg$^2$.

In standard units, $k = 9 \times 10^9$ N-m$^2$/C$^2$.

An alternate constant is often used, namely $\varepsilon_0 = (4\pi k)^{-1}$, so that $\varepsilon_0$ is about $8.8 \times 10^{-12}$ C$^2$/N-m$^2$.

**Always remember that force is a vector. Two or more forces combine as vectors.**

The *magnitude* of the Coulomb force can be written as

$$F = k |Q_1||Q_2|/r^2.$$  

The force is repulsive for like charges, both $+$ or both $-$, and attractive for unlike charges.
An atom of number $Z$ in the periodic table has $Z$ electrons, within a region of about $10^{-10}$ m, and a nucleus about $10^{-14}$ m in size containing $Z$ protons and $N$ neutrons. $A = N + Z$ is about 2 $Z$ for small $A$, but increases to about 2.5 $Z$ for heavy atoms.

$m_e$ is about $9.1 \times 10^{-31}$ kg.

$m_p$ and $m_n$ are about $1.67 \times 10^{-27}$ kg, or about 1800 times $m_e$.

Another fundamental intrinsic scalar property of matter is charge. The unit of charge is the Coulomb. The electron charge $Q_e = -e$, where

$$e = 1.6 \times 10^{-19} \text{ Coulombs}.$$ 

The proton charge is $Q_p = +e$, while the neutron charge is $Q_n = 0$.

An atomic nucleus thus has charge $Q = +Ze$, while an atom has charge $Q = +Ze - Ze = 0$. 

Types of Materials:

- **Insulator**— All electrons are bound to atoms; under ordinary circumstances no charge can be pulled or pushed through an insulator, and charge dumped on the surface just sits there.

- **Conductor**— Each atom, on the average, has lost an electron, and the electrons can migrate through the solid from one end to another if a force is exerted on them. Charge dumped anywhere on the surface distributes to cover the entire surface.

- **Semiconductor**— Under some circumstances the material behaves as an insulator, but it can also be made to behave as a conductor. Thus it functions as a switch, with no moving parts. Semiconductors are the basis of all modern electronics.

- **Plasma**— A plasma is a gas brought to a high enough temperature that electrons are knocked out of atoms. Thus a plasma is like a gaseous conductor, consisting of individual ions and electrons. Most of the normal matter in the universe is in the form of a plasma, but the only plasmas encountered on earth in everyday life are flames.
A charge \((Q_1)\) of 2 \(\mu\)C is at \(x = 0\) and a charge \((Q_2)\) of 1 \(\mu\)C is at \(x = d = 1\) m. Where in the range \(0 < x < d\) can a point charge \(q > 0\) be placed, and feel no force?

The forces on \(q\) will be in opposite directions so we just need to find the \(x\) such that the forces have equal magnitude. The forces are given by \(F_1 = kqQ_1/x^2\) and \(F_2 = kqQ_2/(d - x)^2\). Therefore the forces have the same strength when

\[
Q_1/x^2 = Q_2/(d - x)^2.
\]

Doing the algebra we get

\[
x \sqrt{Q_2} = (d - x) \sqrt{Q_1}, \text{ so}
\]

\[
x = d \left( \frac{\sqrt{Q_1}}{\sqrt{Q_2} + \sqrt{Q_1}} \right).
\]

Plugging in the numbers now results in \(x = 0.586\) m. It makes sense that the point is further from \(Q_1\) than from \(Q_2\) since charge 1 is twice as great as charge 2.
Charge $Q_1$ of 10 $\mu$C is at the origin. Charge $Q_2$ of 20 $\mu$C is at $x = y = 1$ m. Charge $Q_3$ of -30 $\mu$C is at $x = 2$ m, $y = 0$. What are the magnitude and direction of the net force on $Q_2$?

The direction of the two forces can be learned by drawing an accurate diagram and looking at the relevant triangles. The distances between the two pairs of particles are the same, $\sqrt{2}$ m. The magnitudes of the forces are as follows:

$$F_1 = k |Q_1 Q_2|/r_{12}^2 = 0.9 \text{ m, } \theta = 45^\circ.$$

$$F_3 = k |Q_2 Q_3|/r_{23}^2 = 2.7 \text{ m, } \phi = 45^\circ.$$

The net force is given by

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_3,$$

so we get from a simple diagram that

$$F_x = F_1 \cos \theta + F_3 \cos \phi = 2.54 \text{ N}.$$

$$F_y = F_1 \sin \theta - F_3 \sin \phi = -1.27 \text{ N}.$$

In vector notation this is $\mathbf{F} = \hat{i}[2.54 \text{ N}] - \hat{j}[1.27 \text{ N}]$.

Of course the length of $\mathbf{F}$ is $F = \sqrt{F_x^2 + F_y^2} = 2.84 \text{ N}$.

The direction $\mathbf{F}$ makes with the $+x$ axis is given by $\tan \theta' = F_y/F_x$, which leads to $\theta' = -26.6^\circ$. 
Consider a system of 4 fixed point charges, $Q_1$, $Q_2$, $Q_3$, and $Q_4$. Call the force exerted on $Q_1$ by the other three charges $\mathbf{F}_1$, and the force exerted on $Q_4$ by the other three charges $\mathbf{F}_4$. What, if anything, can you say about the relation between these two force vectors?

A  The two force vectors must be equal and opposite, as required by Newton’s 3rd Law.

B  The two force vectors must have the same magnitude, but there would be no relation between their directions.

C  The two force vectors must be equal and opposite in direction, but can have very different magnitudes.

D  There is no relation between the two force vectors; they depend on different quantities.

[The drawing is provided to make the question clear, and does not necessarily depict the forces correctly in any sense.]