Simple Harmonic Oscillator:

acceleration parameter = $-\omega^2$ times displacement parameter.

Example: $a_x = -\omega^2 x$.

Example: $\alpha = -\omega^2 \theta$.

$\omega$ is called the angular frequency, and the period satisfies $T = 2\pi / \omega$.

The general solution to the 2nd Law of motion will be of the form

$$x(t) = A \cos[\omega t + \delta] \text{ or } \theta(t) = \theta_m \cos[\omega t + \delta].$$

Using calculus you can show:

$$v_x(t) = -A\omega \sin[\omega t + \delta], \quad \omega'(t) = -\theta_m \omega \sin[\omega t + \delta],$$

and of course $a_x = -\omega^2 x$, and $\alpha = -\omega^2 \theta$. For rotational oscillators do not confuse the angular speed, which we write as $\omega'$ here, with the angular frequency $\omega$.

The constants $A$ and $\delta$ are closely related to $x(0)$ and $v_x(0)$, since $x(0) = A \cos \delta$ and $v_x(0) = -A \omega \sin \delta$.

Important: The 2nd Law for a simple harmonic oscillator can be solved by either $\cos(\omega t + \delta)$ or $\sin(\omega t + \delta)$, and Quest homework examples may use EITHER form.
Example: The general solution for a simple harmonic oscillator is \(x(t) = A \cos(\omega t + \delta)\). Suppose for a certain oscillator \(\delta = 0\), \(\omega = 10 \text{ sec}^{-1}\), and \(A = 1 \text{ cm}\). Find \(x\), \(v_x\) and \(a_x\) at \(t = 2 \text{ sec}\). [Answers: 0.41 cm, \(-9.1 \text{ cm/s}\) and \(-40.8 \text{ cm/s}^2\).]

A simple harmonic oscillator satisfies

\[
x(t) = [10 \text{ cm}] \cos[(2 \text{ s}^{-1})t].
\]

What is the period of oscillation? [Answer: \(\pi \text{ sec}\).]

How long should a simple pendulum be to have a period of 2 sec? [Answer: about 1 m.]
MASS ON A SPRING:

\[ a_x = -(k/m)x, \]

\[ E = (1/2)mv^2 + (1/2)kx^2 \]

Easy to see that \( v_x = \pm \sqrt{(k/m)(A^2 - x^2)} \).

BASIC SIMPLE HARMONIC OSCILLATOR PARAMETERS:

Amplitude \( A \), \( E = (1/2)kA^2 \).

Angular frequency \( \omega = \sqrt{k/m} \).

Period \( T = 2\pi/\omega \).

Frequency \( f = 1/T \).

General solution of 2nd Law for SHO:

\[ x(t) = A \cos[\omega t + \delta]. \]

Thus \( v_x = -A\omega \sin[\omega t + \delta] \).

\[ a_x = -A\omega^2 \cos[\omega t + \delta]. \]

SIMPLE PENDULUM: \( I\alpha = -mgl\theta \).

Since \( I = ml^2 \), this becomes \( \alpha = -(g/l)\theta \). This is of the SHO form \( \alpha = -\omega^2 \theta \).

So the period will be \( T = 2\pi \sqrt{l/g} \).
"COMPOUND" PENDULUM: $I\alpha = -Mg r_{cm}\theta$. 
Thus $T = 2\pi \sqrt{I/(Mg r_{cm})}$.

**RESONANCE:**

If an oscillator is forced into oscillation by an external driving force of frequency $\omega_0$, where its natural frequency is $\omega$, we find that the amplitude of oscillation increases hugely as $\omega_0$ approaches $\omega$. In fact, with no damping, the amplitude tends to infinity, meaning the oscillator is destroyed.

**DAMPING:**

$$Q = \frac{2\pi E_0}{|\Delta E_{cycle}|}.$$ 
Critical damping: $Q = 0.5$

**MAIN KINDS OF WAVES:** TRANSVERSE or LON-GITUDINAL.

Wave parameters: $A, v_p, \lambda, f$.

Wave function: $y(x,t) = A \sin[kx - \omega t]$.

Relationships: $v_p = \omega/k, k = 2\pi/\lambda, \omega = 2\pi f$.

Note: $v_p = \lambda f$.

For a transverse wave on a string $v_p = \sqrt{I/\mu}$. 
TRANSVERSE WAVES:

\[ y(x, t) = A \sin[kx - \omega t] = A \sin[k(x - v_p t)] \]

for waves moving to the right (along \( +x \)).

Parameters: \( A \), wave amplitude.
\( \lambda \), wavelength.
\( k \), wave number. \( k = 2\pi / \lambda \).
\( \omega \), wave angular frequency.
\( f \), frequency. Note that \( \omega = 2\pi f \).
\( v_p \), phase speed of wave. \( v_p = \omega / k = \lambda f \).

For a transverse wave on a string or wire, \( v_p = \sqrt{T/\mu} \), where \( T \) is the tension in the string and \( \mu \) is the mass per unit length. In general \( v_p \) only depends on mechanical properties of the medium the wave is travelling through.

Note for a wave moving to the left, along \( -x \), the wave function is \( A \sin[kx + \omega t] = A \sin[k(x + v_p t)] \).
TRANSVERSE WAVE PROPERTIES:

REFLECTION:
From a fixed end... a phase change.
From a free end... NO phase change.

SUPERPOSITION:

\[ y(x, t) = y_1(x, t) + y_2(x, t) + \cdots \]

STANDING WAVE ON A STRING:
Superposition of original wave with fixed-end reflections!
Major hint: count the half-wavelengths in the standing wave!!

\[ \text{Example: } L = \lambda + \frac{\lambda}{2} = \frac{3\lambda}{2} \]
\[ \text{so } \lambda = \frac{2L}{3} \]
A simple harmonic oscillator has an angular frequency of 10 rad/sec, an amplitude of 1 cm, and a phase angle of 2. What are its initial position and velocity?

A transverse wave on a string is described by \( y(x, t) = A \sin(kx - \omega t) \). The wave has an amplitude of 1 cm and at \( x = 1 \text{ m} \) at \( t = 1 \text{ sec} \), the string is displaced upward by a distance of 0.4 cm from equilibrium. If the angular frequency of the wave is \( 2 \text{ sec}^{-1} \), what is the wavelength of the wave?

Any function whatsoever of the form \( f(x \pm v_p t) \) satisfies the classical wave equation. This is related to the obvious fact that we can create a wave pulse of any shape whatsoever on any system that can support waves.